Reprinted from
ANNUAL REVIEW OF ASTRONOMY AND ASTROPHYSICS
Vol. 7, 1969

SYNTHETIC-APERTURE RADIO TELESCOPES

By G. W. Swenson, Jr.
Vermilion River Observatory, University of Illinois, Urbana, Illinois

Introduction

Radio astronomy has always been dominated by instrumental problems, especially the need for better angular resolution and sensitivity. While terrestrial optical telescopes are limited in angular resolution by atmospheric inhomogeneity, single-reflector radio telescopes have not yet met this limit and, instead, are limited in both sensitivity and resolution by the enormous costs and formidable engineering difficulties involved in constructing very large reflector antennas. The best angular resolution now available from single-reflector radio telescopes in the range of wavelengths above 5 mm is about 1 min of arc. Few instruments with this performance have been built, and they operate at wavelengths $\sim$1 cm. Better performance is clearly needed, and at longer wavelengths where most of the phenomena of radio astronomy are more easily detectable.

Increased sensitivity can be achieved through longer observing times. However it is not useful to increase sensitivity beyond the point at which more than one source is likely to be within the antenna beam. In many radio astronomical observations this "confusion" effect, rather than the noise in the telescope or its environment, establishes the lower bound for the observable source strength. Thus angular resolution must be improved if fainter sources are to be studied.

Recognizing that the large pencil-beam reflector antenna could not provide the resolution needed for mapping the structure of extended sources or for accurate position measurements on small sources, radio observers made early use of the interferometer (McCready, Pawsey & Payne-Scott 1947). Recognition that an interferometer observing an incoherent source actually measures a term of the Fourier series representing the brightness distribution of the source led to the synthetic antenna consisting of a number of independent interferometers (Stanier 1950).

Impetus to use the interferometer came from the invention of the correlator detector, originally realized in the "phase-switching" interferometer (Ryle 1952), an instrument of great stability and sensitivity.

To obtain higher resolution, radio astronomers have also utilized arrays of antennas, especially at the longer wavelengths. Even with large arrays, however, the costs of achieving resolution comparable with that of even a modest-sized optical telescope are enormous, particularly if pencil-beam patterns are needed. The largest arrays in existence (Mills, Aitcheson, Little & McAdam 1963) produce fan-shaped beams with beamwidths in the narrow dimension of about 1 min of arc.

To produce pencil beams of this size at reasonable cost, one can arrange two long arrays in the form of a cross and with the aid of a correlator multiply the two fan-beams thus produced. The Mills Cross and its relatives, while not aperture-synthesis instruments in the strictest sense of the word, are closely related to the latter both in principle and in application.

THE THEORY OF APERTURE SYNTHESIS

An interferometer system can be expressed schematically in a fairly general way as in Figure 1. Two antennas, each with its amplifying system, are connected to a correlator (or multiplier), which includes an averaging or integrating circuit with a specified time constant that is much longer than the reciprocal of the frequency bandwidth of the system, so that many voltage impulses are averaged in a simple observation.

The interferometer is assumed to observe an extended source of incoherent and statistically stationary radiation. The antennas are pointed in the same direction. For these conditions the output of the correlator is (Swenson & Mathur 1968)

$$r(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{\Gamma}(\xi',\nu)\hat{A}_1(\nu)\hat{A}_2^*(\nu)\hat{G}_1(\xi'-\xi,\nu)\hat{G}_2^*(\xi'-\xi,\nu)e^{j2\pi\nu\tau}d\nu d\xi' \qquad 1.$$

in which

$r(\tau)$ is the output of the correlator
$\hat{\Gamma}$ is the line-integrated brightness distribution of an isolated, finite source
$\hat{A}$ is the frequency response of the amplifier
$\hat{G}$ is the antenna voltage gain
ν is the frequency (Hertz)
$\tau = \tau_g - \tau_i$ is the difference in transit time from a plane wavefront in space to the correlator via the two possible paths

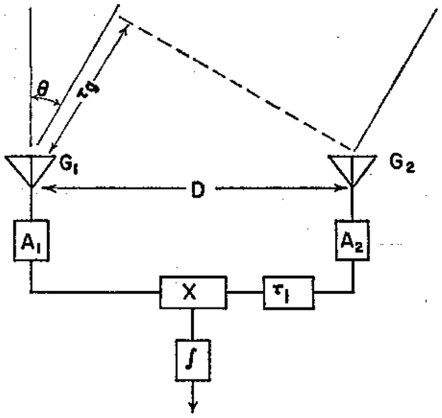

FIG. 1. Basic correlator interferometer system.

τ_g is the geometrical component of τ
τ_i is the instrumental component of τ
$\xi = \sin\theta$

This formula is quite general. In the special case of two identical antennas with identical, band-limited amplifiers it reduces to

$$r(\tau) = \int_{-\infty}^{\infty} d\xi' \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} d\nu \hat{\Gamma}(\xi', \nu) \left|\hat{G}(\xi' - \xi, \nu)\right|^2 \left|A(\nu)\right|^2 e^{j2\pi\nu c} \qquad 2.$$

The time delay τ is the difference between the geometrical delay τ_g and the instrumental delay, τ_1. The instrumental delay is adjusted to the value $D\xi_1/c$, so that

$$\tau = D(\xi' - \xi_1)/c$$

where D is the separation of the antennas in meters and c is the velocity of the wave in space. If the amplifier passband $\Delta\nu$ is sufficiently small, so that the antenna pattern and the brightness distribution do not vary significantly over the band, Equation 2 can be written

$$r(\xi_0, \xi_1, D) = \int_{-\infty}^{\infty} \hat{\Gamma}(\xi', \nu_0) \hat{P}(\xi_0, \xi', \xi_1) d\xi \qquad 3.$$

where ξ_0 is the direction in which the antennas are aimed and ξ_1 is the direction for which $\tau = 0$. The function $P(\xi_0, \xi', \xi_1)$ is the product of the antenna power pattern $\left|\hat{G}(\xi_0 - \xi', \nu_0)\right|^2$, the bandwidth pattern (or delay pattern)

$$B(\xi_1 - \xi', \Delta\nu, D) = \int_{-\Delta\nu/2}^{\Delta\nu/2} \left|\hat{A}(\nu)\right|^2 e^{-j2\pi(\xi_1 - \xi')\nu D/c} d\nu \qquad 4.$$

and the interference pattern

$$F(\xi_1, \xi' D) = e^{-j2\pi(\xi_1 - \xi') D\nu_0/c} \qquad 5.$$

The bandwidth pattern has a peak in the direction ξ_1. When the source and the antenna beamwidth are of small angular extent, the integrand in Equation 4 is nonzero over only a small range of θ centered at θ_0. The instrumental delay can be adjusted to the value $D\xi_0/c$ so the delay pattern also has a peak at ξ. Now let θ' be defined as $\theta_0 - \theta$; then θ is small and

$$\xi' \simeq \sin\theta_0 - \cos\theta_0 \sin\theta = \xi_0 - \xi\cos\theta_0$$

Define u as $D/\lambda_0 \cos\theta_0$ or $D/c \cos\theta_0\, \nu_0$. This is the *spatial frequency* and is the component of the baseline (in wavelengths) in the direction normal to θ_0.

Equation 3 can be rewritten

$$r(u) = \int_{-\infty}^{\infty} \hat{\Gamma}(\xi, \nu_0) \hat{P}(\xi, u, \Delta\nu) d\xi \qquad 6.$$

Now let us examine the form Equation 6 assumes when the bandwidth is narrow enough so that for all baselines the bandwidth pattern is much wider than the antenna pattern, and when the source being observed is, in turn, small compared with the antenna pattern. In this case

$$r(u) = \int_{-\infty}^{\infty} \hat{\Gamma}(\xi, \nu_0) e^{-j2\pi\xi u} d\xi = \hat{\gamma}(u, \nu) \qquad 7.$$

This will be called the "fringe function." It is the Fourier transform of the brightness distribution, and it is apparent, therefore, that the interferometer can be used to make a Fourier analysis of the source structure. This is the basis of aperture synthesis. It is seen from Equation 7 and the definition of u that the spatial frequency measured with a given baseline is the baseline length, in wavelengths, projected on a plane tangent to the celestial sphere at the location of the source. By using a sufficient number of different baselines, enough Fourier components can be measured to permit the reconstruction of the source by Fourier transformation.

It has been assumed that the source is finite, in fact, that it is small compared with the antenna beam. It has been shown by Bracewell (1958) that a source of extent $\Delta\xi$ can be completely represented by sampling its spatial-frequency spectrum at intervals of $u=1/\Delta\xi$. This follows from the basic properties of the Fourier series representation of a function with a finite base. Furthermore, if the smallest detail to be measured is $\Delta\xi_m$, the highest spatial frequency that must be sampled is $U_m=1/\Delta\xi_m$. Thus, the number of baselines needed to perform a complete, one-dimensional analysis on a source is equal to the width of the source divided by the width of the finest detail that is to be resolved. A two-dimensional analysis requires a number of baselines equal to the square of the number for one dimension.

A Fourier series with discrete, uniform spacing of the terms in the frequency domain is a periodic function of the spatial coordinate. If a one-dimensional antenna is synthesized by means of a series of interferometers whose baselines increase successively in length by a uniform interval, the response to a point source is a comb-shaped series of evenly spaced spikes in the ξ dimension. In an actual observation, an isolated, single source can be mapped accurately by this means. If there are other sources present, however, the map of the source under investigation may be seriously distorted by their interactions with the higher-order responses, which are usually termed "grating lobes." The spacing of the responses in the ξ domain is inversely proportional to the increment of the baseline spacing in the u domain; therefore, it is important to plan the observing program according to the nature of the source under investigation. In a two-dimensional synthesis operation, there will be a two-dimensional array of grating lobes, of which examples will be seen.

In the Fourier-series method of aperture synthesis, it is necessary to measure each component of the series only once. If several antennas are available, together with the necessary electronics to permit simultaneous operation of several baselines, the most economical arrangement of the antennas is one which provides the largest number of the necessary baselines with the minimum number of duplications. It is possible to arrange four antennas on a straight line in such a way that there are no redundant baselines; but for larger numbers of elements and for two-dimensional arrays redundancies are inevitable. Several authors (Leech 1956, Bracewell 1966, Moffett 1968) have designed linear, minimum-redundancy correlator arrays with several elements. A linear array discussed by Moffett (1968), for example, has the following configuration: ·8·10·1·3·2·7·8·, where the dots represent the antennas and the numbers represent the number of unit spacing between adjacent elements. All spacings between 1 and 24 exist, plus additional spacings of 31 and 39 units. There are singlefold redundancies at 8 and 31 units. This is a minimally redundant, completely filled array for the resolution corresponding to a maximum baseline of 24 units.

Discussion of the design of synthetic-aperture telescopes is assisted by the flow chart of Figure 2 (Swenson & Mathur 1968), in which the various relationships and transformations among antenna quantities are illustrated. This flow chart applies to the case of incoherent sources or, more generally, to antenna systems linear in power. The right-hand column of the chart represents the more conventional view of antenna theory, in which the brightness distribution of the sky is convolved with the power radiation pattern to obtain the output of the radio telescope. The left-hand column

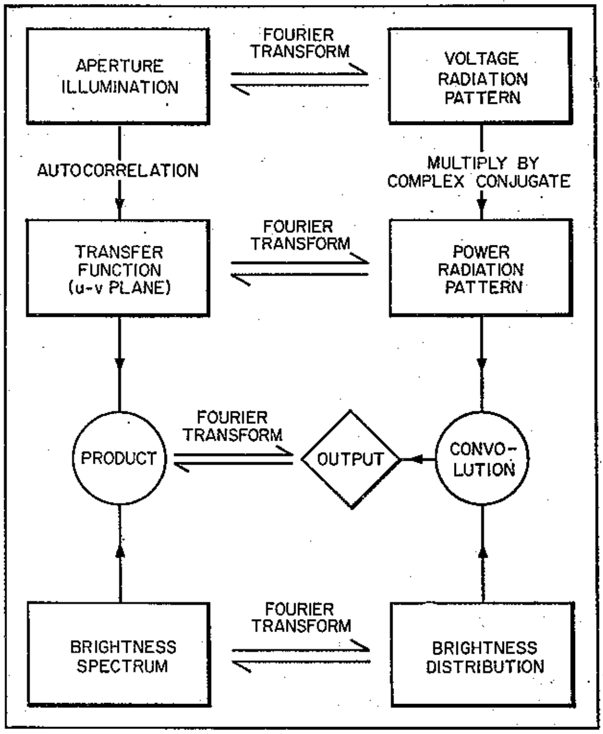

FIG. 2. Relationships among antenna quantities for an incoherent field.

represents an alternative view of antenna theory, more directly applicable to aperture synthesis. The aperture illumination is the distribution of the electric field in the plane of the antenna. In a synthesis array consisting of a number of small antennas, for example, the illumination function would consist of a number of discrete points in the aperture plane. The autocorrelation function of the illumination is called the transfer function. The Fourier transform of the brightness distribution (in spatial coordinates) is the brightness spectrum (in spatial-frequency terms), and the product of the brightness spectrum and the transfer function is the output in terms of spatial frequencies; that is, the *observed* brightness spectrum, whose Fourier transform is the conventional radio telescope output. Clearly, only those spatial-frequency components are present in the output which are also present in the transfer function; thus, the performance of the synthetic telescope can be investigated by examinating its transfer function. The transfer function has the same configuration as the diagram of the antennas in the u dimension, or in the u-v plane in the case of a two-dimensional array.

A simple and very useful graphical method for determining what spatial frequencies are represented in the transfer function of a given correlator array has been described by Bracewell (1961).

In synthesizing a map of a source, the observer may assign any desired weight to each measured spatial-frequency component in order to control the synthesized beam shape and the sidelobes. In this connection, one should distinguish between the sidelobes of the synthetic aperture and the grating lobes caused by the periodicity of the Fourier series. The sidelobes are the usual higher-order diffraction responses of an optical aperture or an aperture antenna; however, in the synthetic-aperture system they are under direct control of the observer. Controlling the weighting of the spatial-frequency components is somewhat analogous to vignetting an optical system or tapering the illumination of an antenna, except that in aperture synthesis the weighting can be done *after* the observations are made, permitting repeated experimenting with weighting functions without the necessity for reobserving.

In general, the sidelobe levels and main-beamwidth are inversely related. Figure 3 shows this relationship for a number of possible weightings of the transfer function. The beamwidth is measured between the first nulls. In general, higher-order sidelobes are successively smaller.

EARLY APPLICATIONS OF APERTURE SYNTHESIS

The first major application of aperture syntheis was made by the Mullard Radio Astronomy Observatory at Cambridge University (Blythe 1957, Ryle & Hewish 1960). A large, pencil-beam interferometer was synthesized at a wavelength of 1.7 m (P. F. Scott et al. 1961), as illustrated in Figure 4. The synthetic interferometer comprised an east-west array and a north-south array, whose centers are separated by a distance considerably greater than the length of either element. It is, in effect, a Mills Cross whose arms

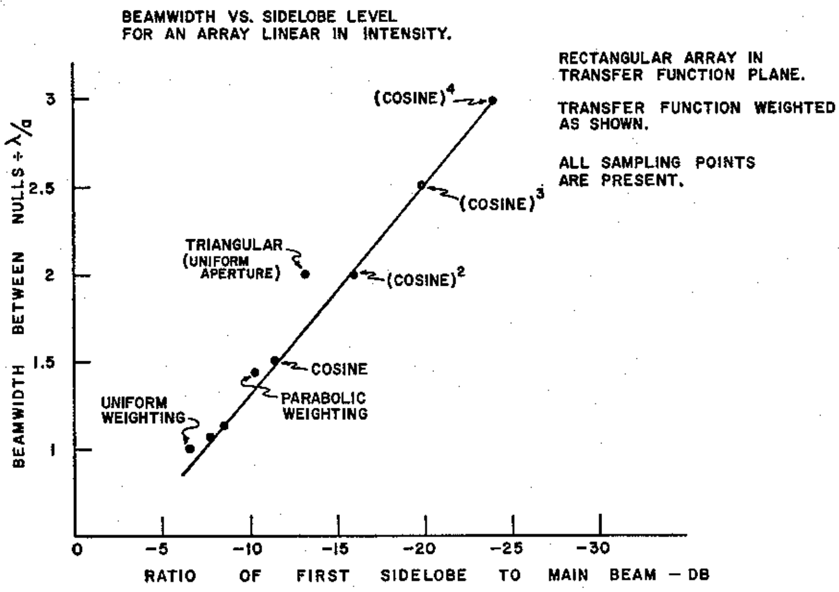

FIG. 3. Relationship among beamwidth, sidelobe level, and transfer-function weighting.

are displaced from one another so that the pencil-beam response is multiplied by a sinusoidal interference pattern whose period is shorter than the beamwidth. In practice, the east-west arm is a long parabolic cylinder, tiltable about an east-west axis to adjust its declination pointing. The north-south arm, however, uses a short segment of parabolic cylinder mounted on rails, so that it can assume successive positions along the north-south arm. Observations are made with the short segment in the first position, and the output of the correlator is stored on a computer tape. Then the short segment is moved to another position and the observations repeated, and the process is continued until the desired right-ascension range has been observed with all positions of the short segment.

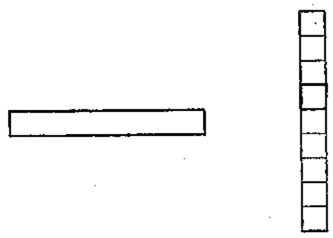

FIG. 4. Synthetic interferometer of the University of Cambridge.

Following the completion of a series of observations, the data are added in phasor form to obtain the synthesized result. For any given right ascension, the result is a map of a north-south strip of sky one-half of a degree wide in right ascension and about 5° wide in declination (the beamwidth of the synthetic arm), and weighted by the synthetic beam pattern. This weighting can be removed after the synthesis is complete.

The instrument described above was used to produce the well-known 4C survey (Gower et al. 1967, Pilkington & Scott 1965).

Two other synthesis instruments have been built at Cambridge. One operates at 38 MHz (Costain & Smith 1960) and produces a synthetic pencil beam 1° in diameter. It has been used for mapping the Galaxy and for investigating a number of galactic objects (Kenderdine 1963). Another operates at 178 MHz and uses the east-west arm of the synthetic interferometer, together with a smaller element borne on a rail track which joins the east-west arm approximately at right angles (Crowther & Clarke 1966). It provides a synthesized beam at the zenith which is 24 min of arc in right ascension and 19 min in declination.

In the operation of a synthesis instrument of this type, in which only one dimension is synthesized, the time needed to map a given area is proportional to the number of positions of the movable element. No part of the map contained within the primary beam pattern of the movable element can be constructed until observations have been completed for all positions of the element. If both arms of the cross were to be physically constructed, each single observation would be complete in itself and the map could, in theory at least, be constructed as the observations proceeded. However, the total time needed to map an area of sky as large as, or larger than, that subtended by the beam of the movable antenna is the same in both cases.

There is no reason why two or more movable elements cannot be used simultaneously, provided the duplicate electronic systems are available. This speeds up the observing in proportion to the number of elements. In the limit in which all elements are present, each with its own amplifiers, correlator, and recorder, the entire declination range within the beamwidth of *one* of the small antenna elements can be mapped simultaneously. This requires as much antenna structure as would a conventional antenna array whose beamwidth equals the synthesized beamwidth, plus n times as much electronics, where n is the ratio of the single-element beamwidth to the synthesized beamwidth. The benefit from the additional investment in electronics is that the mapping proceeds n times faster.

A disadvantage of the conventional array, in which the contributions of the individual antennas are added before being detected, is that most of the information received by the antennas is rejected by the system. One way to utilize more of the available information is the simultaneous synthesis procedure described above; another is the provision of a number of separate beam-forming networks within the array (Mills et al. 1963, Wild 1967). The two methods are essentially equivalent with respect to available performance and required amounts of electronic equipment, but the synthesis method does have the advantage that a number of different aperture illumination functions can be used on the same data without reobserving, and the disadvantage that electronic computing facilities are required.

The effective collecting area of a synthetic-aperture telescope may be derived from the fact that the noise components of the separate baselines are independent. This is true even though the observations on different baselines may be made simultaneously and may share a common antenna. When the outputs of the correlators are added, the source-contributed components add numerically while the noise components add as the square root of the sum of the squares. The effective area therefore varies as the square root of the number of baselines in the synthetic array. From Equation 1 it is seen that the effective area of a baseline is the product of the effective areas of the individual antennas; from this the effective area of the synthetic array is readily calculated for any given case.

Supersynthesis

The Cambridge aperture-synthesis telescopes are meridian-transit instruments, suitable for mapping large areas of the sky in a systematic way. Such instruments are less suitable for mapping selected sources or areas in great detail. For one thing, very high resolution requires very large synthetic instruments, and it is necessary to reduce the observing time or the size of the antenna structure, or both, if even synthetic-aperture systems are to be practicable for resolutions of the order of a second of arc. For another, it is demonstrably important to be able to map selected sources in such detail without suffering the lengthy periods of observing that would be necessary with a transit telescope. Fortunately, another synthesis technique, again pioneered by the Cambridge group (Ryle 1962, Elsmore et al. 1966), is available which makes use of the rotation of the Earth in a different way to accomplish aperture synthesis.

Supersynthesis, or Earth-rotation synthesis, makes use of the fact that the projection of a baseline on a particular part of the celestial sphere is rotated and foreshortened as the Earth turns with respect to the sky. During a day, therefore, a single terrestrial baseline assumes many different values of spatial frequency with respect to a given source. By tracking a source for a substantial part of a day with the antennas comprising a terrestrial baseline, a substantial reduction can be effected in the number of antennas, or the number of moves of a set of antennas, necessary to accomplish the synthesis of a given aperture.

Figure 5 shows the geometry of an interferometer. The baseline intersects the celestial sphere at B, which has the declination d and the local hour angle h. The source is at point S, with coordinates δ and H. The projection of the baseline on the intersection of the plane SOB and a plane tangent to the celestial sphere at S is $D \cos \theta$. Resolving this into components in the declination and hour-angle directions gives

$$u = D \cos d \sin (H - h)$$

$$v = D[\sin d \cos \delta - \cos d \sin \delta \cos (h - H)]$$

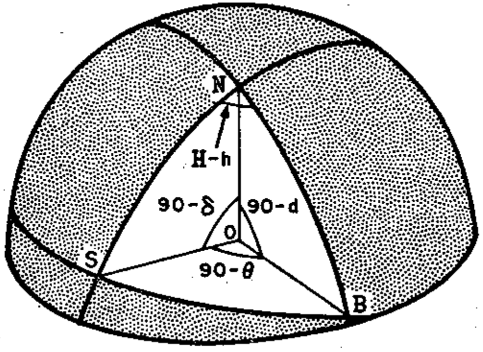

FIG. 5. The geometry of an interferometer.

These are the parametric equations of an ellipse, the locus of a vector in the *u-v* plane which represents the values of spatial frequency for the given baseline. It is instructive to examine some limiting special cases. Assuming that the baseline is east-west and horizontal, observing a source at the celestial pole the *u-v* plane locus is a circle of radius D, the baseline length in wavelengths. For a source on the equator the locus is a straight line of length $2D$.

The *u-v* plane locus of a single baseline is the transfer function (Figure 2) of the interferometer. If several baselines are available, either simultaneously or sequentially, the transfer function is the superposition of all their loci.

The first instrument to use this technique was the 178 MHz interferometer of the University of Cambridge (Ryle & Neville 1962) described above. Portions of the east-west arm of the instrument were used in combination with the movable element and a small additional antenna to provide a total of 75 contiguous spacings along an east-west line 600 wavelengths long. As the elements of the interferometer can be directed only along the meridian, only the region within 4° of the north pole could be mapped. An angular resolution of 4′.5 was obtained with this baseline. Subsequently, a similar map was made of the north polar region at 81.5 MHz with a different instrument (Branson 1967) with a maximum baseline of 250 wavelengths. This survey extends 40° from the pole and has an angular resolution of 10′.

Following demonstration of the feasibility of the Earth-rotation synthesis technique, the Cambridge group built a more versatile instrument, capable of mapping individual sources at shorter wavelengths. The One-Mile Radio Telescope (Ryle 1962, Elsmore et al. 1966) consists of three mechanically steerable paraboloids, each 18 m in diameter, along an east-west baseline. Two of the telescopes are fixed at the ends of a 750-m baseline, and the third is movable on rails to extend the baseline to a total of 1550 m. The telescopes are of the Blaw-Knox Co.-Howard Tatel equatorial design, probably usable to 3 or 5 cm wavelength. It has been used at frequencies of 408 and 1407

MHz where it can achieve circular beamwidths (at the pole) of 1′.3 and 0′.4 respectively. The beamwidth in declination varies as the cosecant of the declination of the source, in accordance with Equation 9. The instrument has been used for the survey of sample regions of the sky, in order to extend the statistics of source intensities to very weak sources (Pooley & Kenderdine 1968), and the mapping of selected objects (Ryle, Elsmore & Neville 1965a, b; Macdonald, Kenderdine & Neville, 1968). Figure 6 is an example of the kind of map that can be made with the One-Mile Telescope at 1407 MHz.

TECHNICAL PROBLEMS OF APERTURE SYNTHESIS

The aperture-synthesis technique has been well demonstrated by the Cambridge workers, and the method will undoubtedly form the basis for extensive future work in high-resolution radio astronomy. The best resolution achieved to date in complete synthesis work has been about 0′.4, but the method is undoubtedly workable for resolution at least an order of magnitude better. In the extension of the technique to second-of-arc resolution, however, some difficult technical problems will be encountered, with respect both to electronics and to the inhomogeneous atmosphere of the Earth.

The electronic system.—Figure 7 is a simplified block diagram of a single interferometer. Of the equipment shown schematically, the preamplifier, mixer, delay lines, and *i-f* amplifier must be provided for each antenna. One central local oscillator sends its signal to all mixers in the array and one delay computer controls all the delay lines. A correlator and a recording channel must be provided for each simultaneously operating baseline. The largest possible number of simultaneous baselines is $N(N-1)/2$ where N is the number of antennas. Thus, the number of correlators and recorder channels varies approximately as N^2, while the other electronics vary directly as N.

The preamplifier must be mounted as close as possible to the focal point of its antenna, to minimize transmission-line losses, and it must have the lowest possible noise temperature if the best obtainable sensitivity is to be achieved. Current practice is to use solid-state parametric amplifiers, with which system noise temperatures in the range 50° to 100° K can be obtained. It is necessary that the preamplifier not introduce variable phase shifts large enough to affect the accuracy of the measurement. As the variable phase shift in the entire system should be restricted to, say, 10°, each component must contribute considerably less than that. Solid-state paramps appear to meet this requirement without difficulty. Similarly, the heterodyne mixers are of conventional type.

The local oscillator system, however, is a critical component of the aperture-synthesis system. The mixer multiplies the radio frequency signal of finite bandwidth by the sinusoidal local-oscillator signal. The result is that the signal band is translated both upward and downward in frequency; normally the lower-frequency band is selected for amplification in the in-

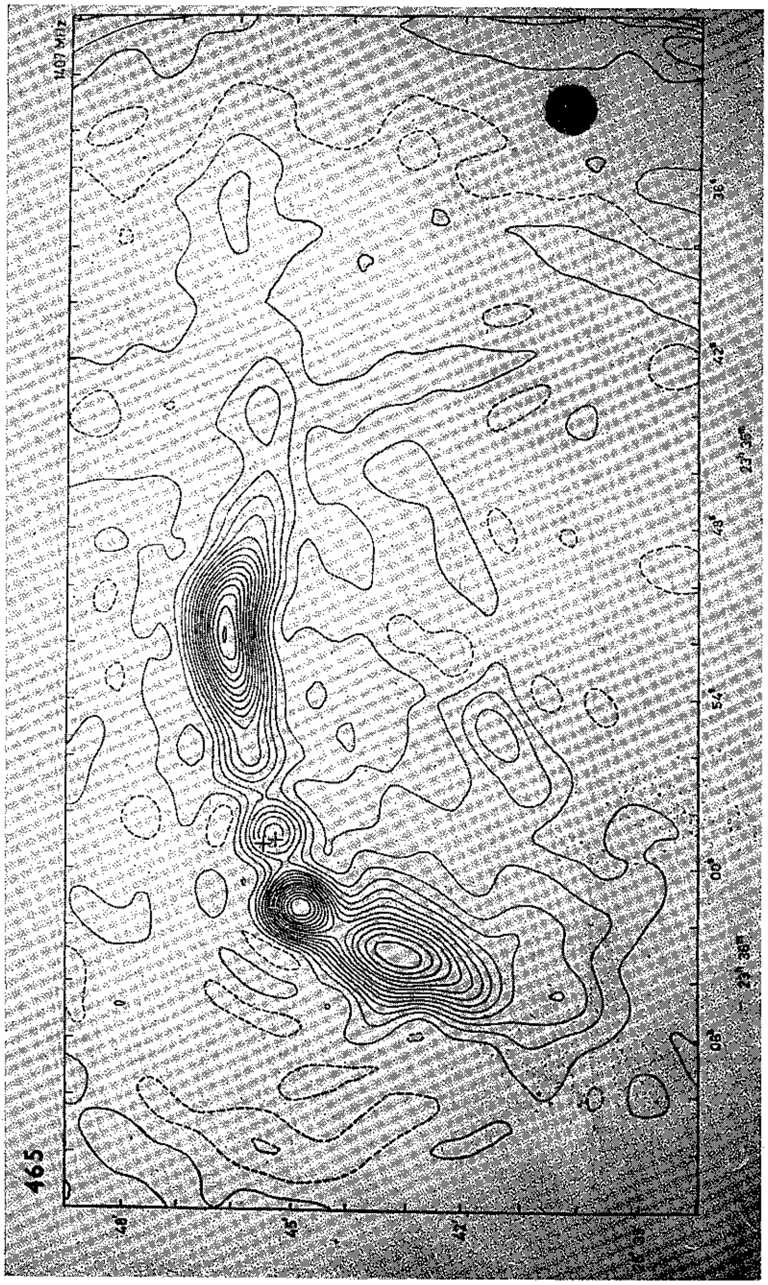

FIG. 6. Map of 3C465 made with the Cambridge One-Mile Radio Telescope (Macdonald, Kenderdine & Neville 1968).

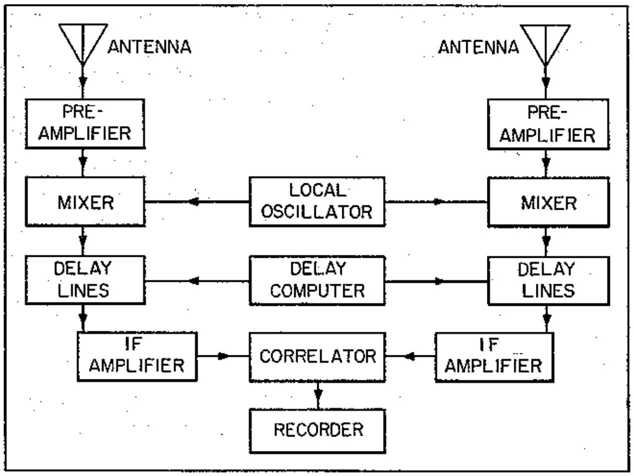

FIG. 7. Simplified block-diagram of a correlator interferometer.

termediate-frequency amplifier. The information content of the signal band is faithfully preserved in the frequency conversion process, provided the local-oscillator signal is truly monochromatic; but the phase of the intermediate-frequency signal depends upon the phase of the local-oscillator signal. In order that the *i-f* signals reaching the correlator have the same mutual phase relationships as they do in the radio frequency bands, the local-oscillator signals injected into both mixers must be of exactly the same frequency and phase. A constant phase difference can be compensated for in the postcorrelation processing, but no time variation of the relative local-oscillator phase can be tolerated.

The distance between antennas can be large, ~10 km for second-of-arc resolution at decimeter wavelengths. Preservation of phase relationships over these distances is difficult, whatever transmission medium is employed. If the local-oscillator signal is radiated through space from a central location to the antennas, the variable refractivity of the atmosphere and reflections from moving objects (including the telescopes) may cause intolerable phase variations. A more reliable method is to transmit the signal through coaxial cables, though even here it is necessary to take special precautions. To combat losses in the cable, the reference signal is generally transmitted at a low frequency and the desired local-oscillator signal is produced by a frequency multiplier at the antenna. A low-loss cable generally employs a gas dielectric, whose phase velocity may vary with ambient atmospheric conditions. Burying the cable 1 or 2 m in the ground restricts the temperature variations to a very slow rate. The gas in the cable should be kept dry and under a constant absolute pressure. By these means it is possible to achieve adequate phase stability for aperture synthesis at, say, 10″ resolunature of the observational technique suggests that greater versatility is desirable. If too few antennas are available to yield the necessary set of spatial-frequency components with one array configuration, it is, of course, essential that the antennas be movable so that the necessary complementary configurations can be set up.

The angular resolution obtainable with conventional optical telescopes is limited by the inhomogeneous refractivity of the Earth's atmosphere to about $0\overset{''}{.}5$. While the lower limit of detectable optical flux density can be decreased by increasing the exposure time, no improvement occurs in the angular resolution, as all contributions to the image are positive in sense. The output of the correlator interferometer is either positive or negative, depending upon the phase difference between the waves arriving at the two antennas; random perturbations in output caused by varying atmospheric refractivity can be reduced by averaging a number of observations, provided the individual fluctuations in relative phase are less than 90°. It is apparent, then, that by making repeated observations with all necessary baselines, an aperture can be synthesized with any desired resolution, provided that the variations in refractivity are not too extreme.

Little is known concerning the random component of the refractivity of the atmosphere. Successful results of the Cambridge One-Mile Radio Telescope and of the three-element interferometer of the National Radio Astronomy Observatory in West Virginia (1967) demonstrate that the atmosphere is stable enough to permit synthesis observations at resolutions of 24″ and 8″, respectively.

The refractivity of the atmosphere at decimeter wavelengths depends mainly on its water content. It is to be expected, then, that the phase fluctuations should also depend on the water content, and the experiences with the NRAO instrument tend to confirm this. The most appropriate site for a decimeter synthesis instrument of high resolution is, therefore, one in which the atmospheric water-vapor content is low and the air relatively stable and free of weather fronts.

While the water-vapor structure of the troposphere determines the performance of an interferometer in the decimeter-wavelength range, the performance in the dekameter-wavelength range would undoubtedly be determined by ionospheric structure. Apparently no systematic study has been made to determine the atmospheric limitations on long-baseline operation at these wavelengths, and it cannot be predicted at present what the limits of achievable resolution are in either the dekameter range or the meter-wavelength range in which both ionospheric and tropospheric effects are probably important.

THE DESIGN OF SUPERSYNTHESIS ARRAYS

The instruments presently in operation which are capable of supersynthesis are linear arrays of two or three antennas, one or two of which can be moved along the line of the array to achieve a number of baseline lengths.

Particularly in the case of the east-west array, the design problem is fairly simple. The desired angular resolution is first specified, which establishes the maximum baseline length. Then the desired "field of view" (distance between grating lobes) is specified, which establishes the shortest baseline needed. The ratio of the longest to the shortest baseline length is the number of baselines needed. In general, as discussed earlier, there will be unavoidable redundancies in certain baseline lengths, though arrays with minimal redundancies can be designed. For a source at the celestial pole the transfer function is a set of concentric circles, one circle for each different baseline. For sources of different declinations the loci are coaxial and are symmetrical with respect to the u and v axes. The minor axes of the ellipses vary as the sine of the declination, so that the loci are straight lines along the u axis. The east-west array is therefore incapable of pencil-beam synthesis on equatorial sources, and has severely degraded performance at low declinations.

If the axis of the linear array has an azimuth other than 90°, the u-v plane loci for a source at the equator are a set of straight lines parallel with the u axis. In general, the linear array is not satisfactory for equatorial sources and its use for pencil-beam synthesis is restricted to sources above, say, 30° declination.

Analysis of the transfer functions of supersynthesis arrays with arbitrary baseline azimuths is very time consuming, particularly if the antennas have hour-angle limits that vary with declination, or if the view of the sky is limited by an irregular horizon. If the array is two-dimensional, computer programs have been used (NRAO 1967, Mathur 1969) for extensive investigations of array configurations. Figure 8a is the transfer function of the three-element interferometer at the National Radio Astronomy Observatory (Mathur 1969) observing a source at the north pole. The baseline lies on an azimuth of 63°. Sixteen collinear baselines are available, three at a time, with lengths ranging from 900 to 25 000 wavelengths. The elliptical appearance of the diurnal loci is caused by the computer printer format; the loci are actually circular. Only one half of the u-v plane is represented. As a baseline has a direction but no positive or negative sense, the transfer function is symmetrical through the origin. In the figure the numbers represent sampled points in the u-v plane while the dots represent unsampled points. As is seen in Figure 2, the power-reception pattern is the Fourier transform of the transfer function. This is computed by the "fast Fourier transform" algorithm (Cooley & Tukey 1965) and plotted as Figure 8b. The power received by the array from a point source is plotted as a vertical deflection of a line at a point whose horizontal coordinate is the hour angle of the source and whose vertical coordinate is the declination. The circular "ripples" in the pattern are the grating lobes caused by the coarse spacing of the diurnal loci in the transfer function. The declination interval between successive graphs of the power level is $4''.7$. This particular beam pattern was derived by applying a Gaussian taper to the transfer function, decreasing at the edge of the transfer function to −15 db of its value at the center. A beam pattern

Another supersynthesis instrument is under construction at Westerbork, The Netherlands (Casse 1968). It consists of twelve 25-m, equatorially mounted paraboloids on an east-west baseline. Ten of the antennas are fixed to the ground at intervals of 144 m, and the other two can move along a railway 300 m long at one end of the fixed array. Observations can be made with the movable antennas at arbitrary points along the track, so that any desired field-of-view can be obtained. The antennas are designed to track a source for 12 hr ($\pm 6^h$ from the meridian) per day, and can observe at any declination accessible from the site. Correlators are to be provided for each combination of one fixed and one movable antenna. It is intended to observe at two polarizations simultaneously. Furthermore, observations will be made simultaneously on two orthogonal polarizations for each antenna, and correlators will be provided for all cross-polarized as well as all co-polarized combinations, so that a total of 160 correlators is needed. Five 12-hr periods are needed to map a source with a half-power beam of $22'' \times (22''$ cosec δ, at a wavelength of 21 cm, using the maximum available baseline of 1620 m. It is anticipated that the minimum detectable flux density will be 2×10^{-29} $\mathrm{Wm^{-2}\,Hz^{-1}}$. Observations with this instrument will commence in 1969.

The University of Sydney is in the process of converting its grating-cross telescope at Fleurs, New South Wales from the Mills Cross mode of operation to the supersynthesis mode (Christiansen & Wellington 1966). The instrument comprises two linear arrays, one north-south and one east-west, each including 32 equatorial paraboloids 7.5 m in diameter spaced uniformly by 12.2 m and two 13.7 m paraboloids spaced 18.3 m and 410 m from the ends of the array, respectively. Each large antenna is correlated with each of the small antennas, so that 64 correlators are required for each array of the cross. Either arm of the cross could be used for pencil-beam synthesis of sources in polar regions, but in order to make equivalent observations of equatorial sources, individual antennas would have to be correlated *between* the two arms. This is the only correlator array known to be under construction with which pencil-beam observations on low-declination sources could be made.

At Stanford University a linear, correlator-array telescope is under construction (Bracewell 1966) having five 18-m equatorial paraboloids designed to operate at a wavelength of 2.8 cm. The antennas are fixed to the ground in a minimum-redundancy configuration with a unit spacing of 22.9 m, giving a field of view of $4'.2$, and an overall length of 205.7 m, giving a minimum beamwidth of $17''$. This array is to operate at a shorter wavelength than any other known to be in the planning stage.

CONCLUSION

The correlator array operating in the supersynthesis mode appears to offer the best prospect for achieving high resolution and high sensitivity at radio wavelengths. It appears that the Earth's atmospheric properties will permit operation at resolutions of $1''$ or better; in fact, the ability to reobserve individual Fourier components of a source and to improve these data by averaging repeated observations suggests that the atmosphere may not provide the ultimate limit on resolution in radio astronomy as it does at present in optical astronomy.

Although aperture synthesis is still a new technique, with active observational programs at only one or two observatories, the technique has been well investigated and the feasibility of substantial improvements in sensitivity and resolution has been demonstrated. Within the next two or three years, several new supersynthesis instruments are expected to come into operation, so that at least the strong sources at declinations above ±30° should soon be mapped to resolutions of 10″ or 20″ at several wavelengths.

ACKNOWLEDGMENT

The author expresses his appreciation for their assistance to Drs. N. C. Mathur and G. H. Macdonald, to Professors W. N. Christiansen and Sir Martin Ryle, and to Ir. J. L. Casse.

LITERATURE CITED

Blythe, J. H. 1957, *MNRAS*, **117**, 644–51

Bracewell, R. N. 1958, *Proc. Inst. Radio Engrs.*, **46**, 97–105

Bracewell, R. N. 1961, *Trans. Inst. Radio Engrs.*, **AP-9**, 59–67

Bracewell, R. N. 1966, *Progress in Scientific Radio, XV General Assembly of URSI*, 243–44 (Natl. Acad. Sci. Publ. 1468, Washington, D.C.)

Branson, N. J. B. A. 1967, *MNRAS*, **135**, 149–74

Casse, J. L. 1968 (Private communication)

Christiansen, W. N., Wellington, K. J. 1966, *Nature*, **209**, 1173–76

Cooley, J. W., Tukey, J. W. 1965, *Math. Comput.*, **19**, 297–301

Costain, C. H., Smith, F. G. 1960, *MNRAS*, **121**, 405–12

Crowther, J. H., Clarke, R. W. 1966, *MNRAS*, **132**, 405–12

Elsmore, B., Kenderdine, S., Ryle, M. 1966, *MNRAS*, **134**, 87–95

Gower, J. F. R., Scott, P. F., Wills, D. 1967, *Mem. RAS*, **71**, 49–144

Kenderdine, S. 1963, *MNRAS*, **126**, 41–54

Leech, J. 1956, *J. London Math. Soc.*, **31**, 160–69

Macdonald, G. H., Kenderdine, S., Neville, A. C. 1968, *MNRAS*, **138**, 259–311

McCready, L. L., Pawsey, J. L., Payne-Scott, R. 1947, *Proc. Roy. Soc. A*, **190**, 357

Mathur, N. C. 1969, *Radio Sci.*, **4**, No. 3

Mills, B. Y., Aitcheson, R. E., Little, A. G., McAdam, W. B. 1963, *Proc. Inst. Radio Engrs. Aust.*, **24**, 156–65

Moffet, A. T. 1968, *Trans. Inst. Elect. Electron. Engrs.*, **AP-16**, 172–75

National Radio Astronomy Observatory. 1967, *A Proposal for a Very Large Array Radio Telsceope—I: The VLA Concept* (Clearinghouse for Fed. Sci. and Tech. Inf., Springfield, Va., Stock No. PB 173, 912)

Pilkington, J. D. H., Scott, P. F. 1965, *Mem. RAS*, **69**, 183

Pooley, G. G., Kenderdine, S. 1968, *MNRAS*, **139**, 529–50

Read, R. B. 1963, *Ap. J.*, **138**, 1–29

Ryle, M. 1952, *Proc. Roy. Soc. A*, **211**, 351–59

Ryle, M. 1962, *Nature*, **194**, 517

Ryle, M., Elsmore, B., Neville, A. C. 1965a, *Nature*, **205**, 1259

Ryle, M., Elsmore, B., Neville, A. C. 1965b, *Nature*, **207**, 1024

Ryle, M., Hewish, A. 1960. *MNRAS*, **120**, 220–30

Ryle, M., Neville, A. C. 1962, *MNRAS*, **125**, 39–55

Scott, P. F., Ryle, M., Hewish, A. 1961, *MNRAS*, **122**, 95–111.

Stanier, H. M. 1950, *Nature*, **165**, 354–55

Swenson, G. W., Jr., Mathur, N. C. 1967, *Proc. Inst. Radio Electron. Engrs. Aust.*, **28**, 370–74

Swenson, G. W., Jr., Mathur, N. C. 1968, *Proc. Inst. Elect. Electron. Engrs.*, **66**, *No. 12*

Thompson, M. C., Jr., Wood, I. E., Smith, D., Grant, W. B. 1968, *Trans. IEEE*, **AP-16**, *No. 6*

Wild, J. P. 1967, *Proc. Inst. Radio Electron. Engrs. Aust.*, **28**, 279–91