

# 21cm Instrument Simulation Software and Foreground Subtraction

Dave McGinnis

# 3-D Radio Maps

- With the 3-D radio intensity map, we will pull the unique BAO signal out of a map that is dominated by foregrounds.
- Foregrounds consist of galactic synchrotron emission, point radio sources, etc.
- Foreground subtraction will be one of the most difficult parts of the project and will dominate the design of the instrument
- To tackle foreground removal, we are developing a large package of software to simulate the telescope and test foreground removal

# Sky Simulation Software

- Simulate telescope response
  - Include the effects of noise and telescope errors
  - Provide data stream that would mimic the real telescope
- Reconstruct the sky temperature from telescope response
- Foreground removal algorithms

# Code Suite

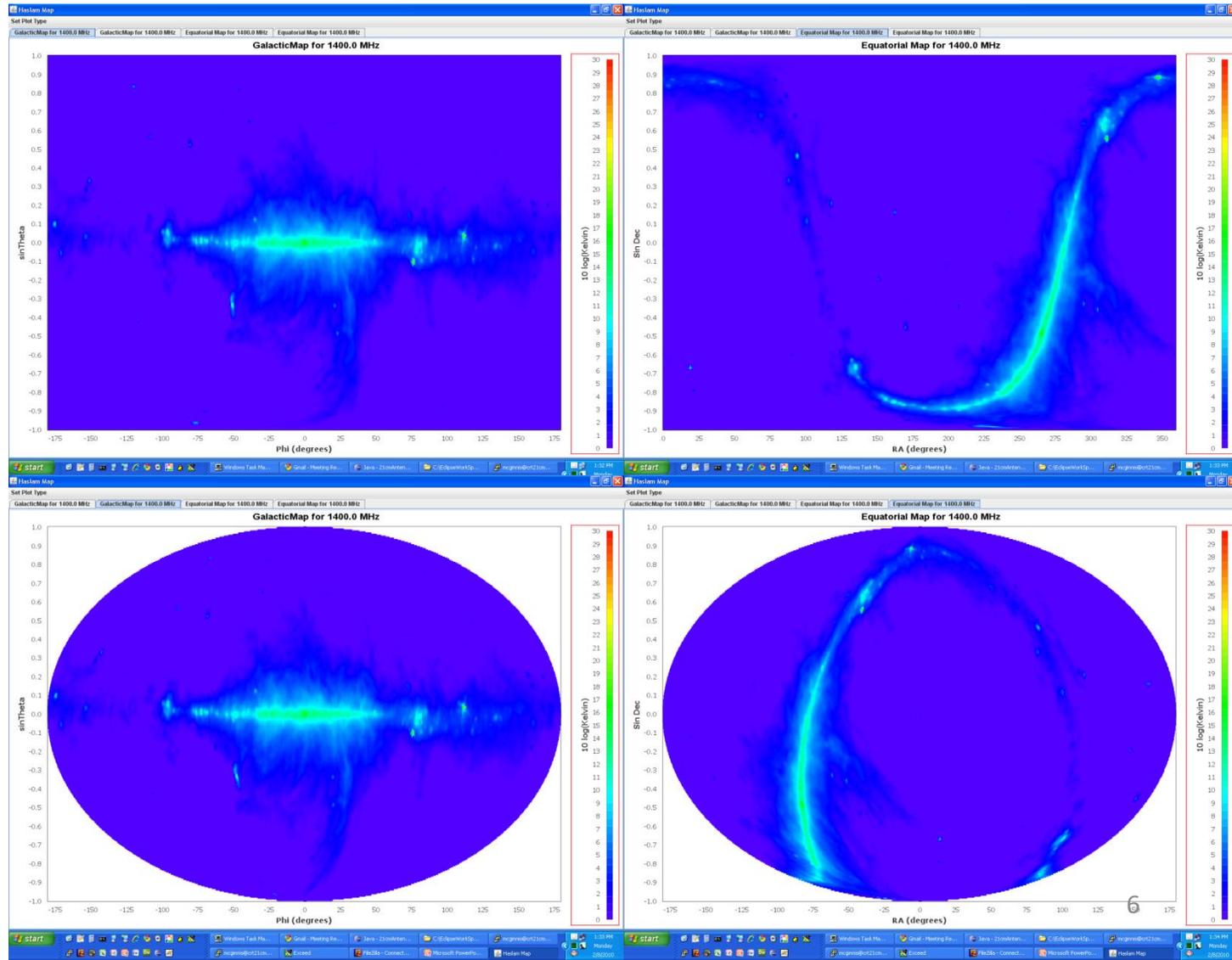
- 29 Java classes organized into 6 packages
- Major Packages
  - Sky Map Generator
  - Cylinder Visibility Simulator
  - Cylinder Visibility Modeler
  - Sky Reconstructor
  - Foreground Removal Algorithms

# **SKY MAP GENERATOR**

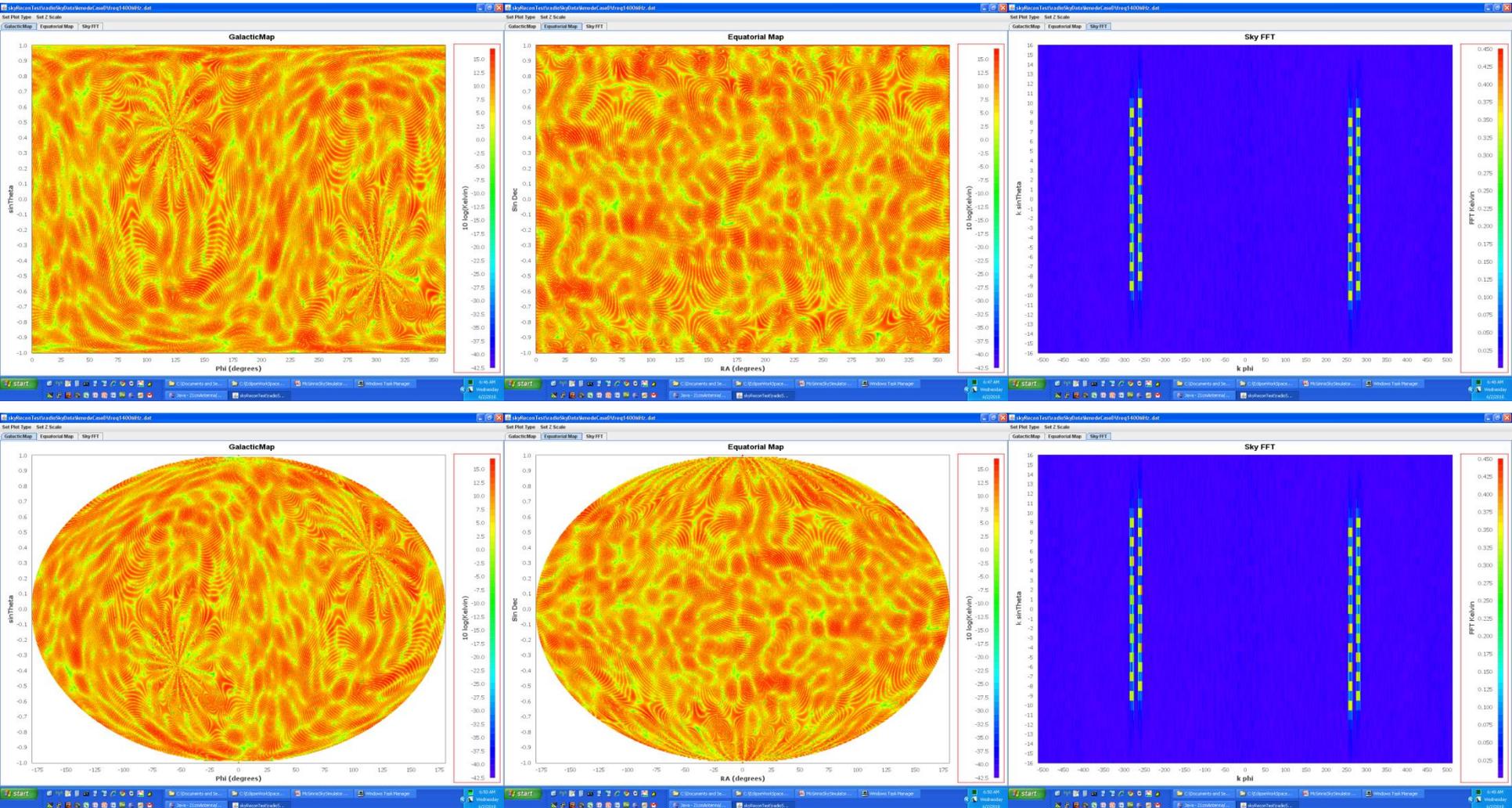
# Sky Map Generator Plotter for Haslam

## Sky Map at 1.4 GHz

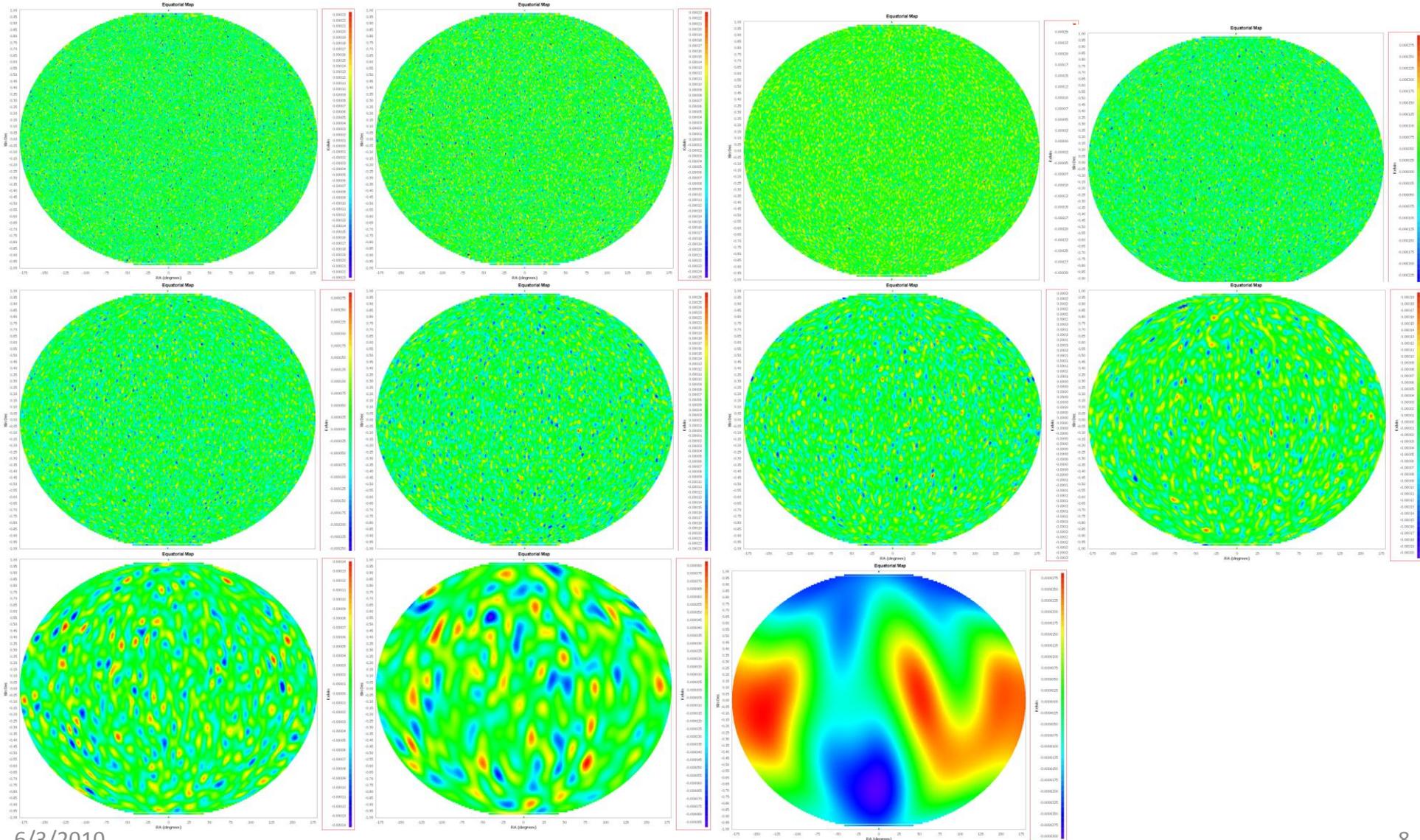
- Maps in Healpix format
  - $N_{\text{side}} = 512$
- Maps use MIT Angelica 10 parameter frequency fit
- Maps are about 100MB in size



# Designer Skies



# BAO Signal First Peak from 400-1400MHz



# CYLINDER VISIBILITY SIMULATOR

# Cylinder Visibility Formulation

## Formulation of Cylinder Visibilities

Dave McGinnis  
November 5, 2009

### **Feed Amplitude**

The voltage received at a feed located at coordinate  $\mathbf{r}$  is:

$$\frac{v(\vec{r})}{\sqrt{2R}} = \iint_{\Omega} f(\Omega) a(\Omega) e^{-j\vec{\beta}(\Omega) \cdot \vec{r}} d\Omega \quad (1)$$

The sky flux amplitude is:

$$|f(\Omega) d\Omega|^2 = \frac{kT_{sky}(\Omega)}{\lambda^2} d\Omega_{pow} \quad (2)$$

where  $d\Omega_{pow}$  is the differential power solid angle area. The incoming wave vector is:

$$\vec{\beta}(\Omega(\theta, \phi)) = \frac{2\pi}{\lambda} (\sin(\theta)\hat{x} + \cos(\theta)\sin(\phi)\hat{y}) \quad (3)$$

The collecting area of the feed is:

$$A(\Omega) = |a(\Omega)|^2 \quad (4)$$

The noise power generated by the feed amplifier is:

$$P_z = |p_z|^2 \quad (5)$$

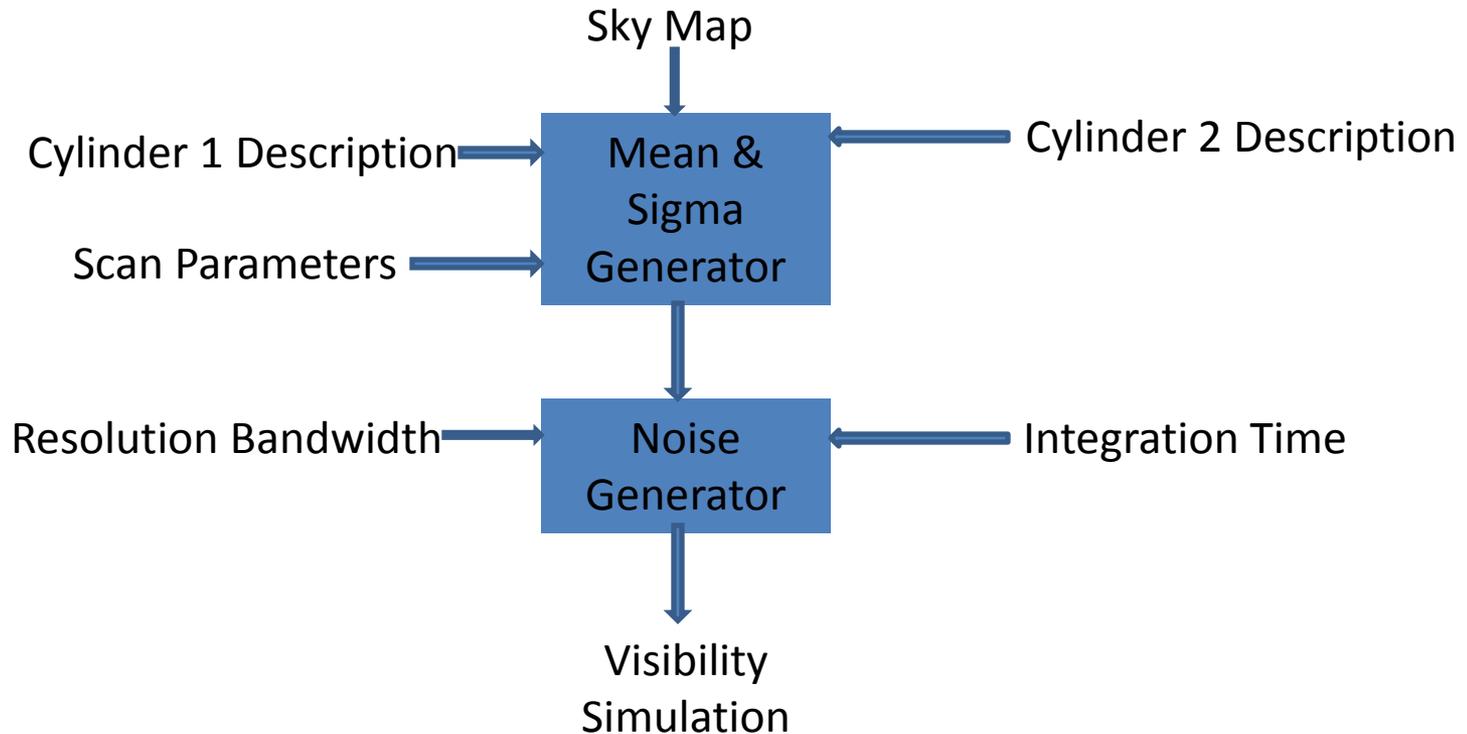
If the sky is pixelized into  $\mathbf{q}$  pixels then, the signal amplitude at feed  $\mathbf{n}$  of cylinder  $\mathbf{m}$  is:

$$p_{n,m} = p_{z_{n,m}} + \sum_q \Delta\Omega_q f_q a_{q,n,m} e^{-j\vec{\beta}_q \cdot \vec{r}_{n,m}} \quad (6)$$

### **Cylinder Amplitude**

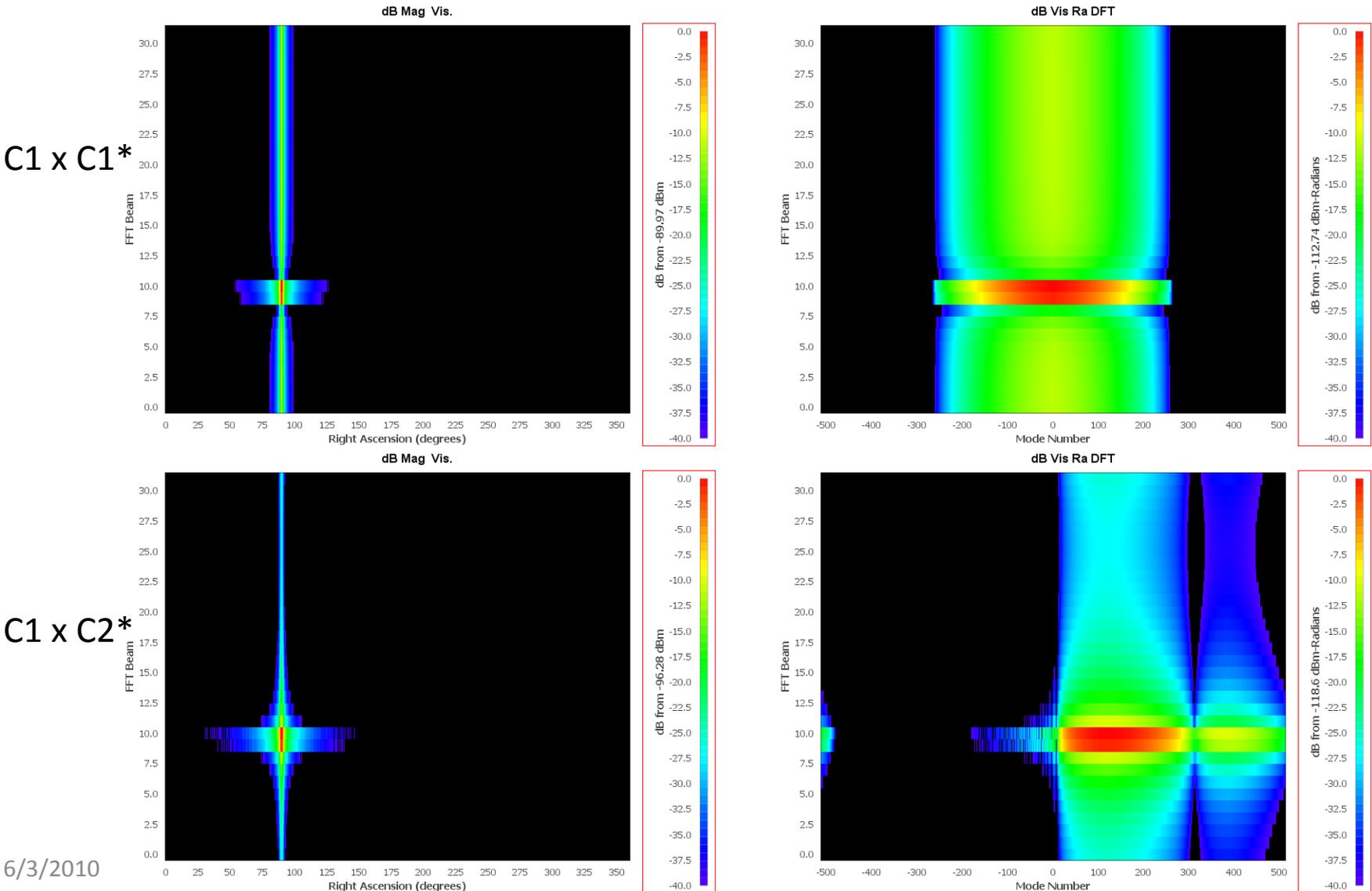
A spatial Fourier transform will be taken of the cylinder feed voltage.

# Cylinder Visibility Simulator



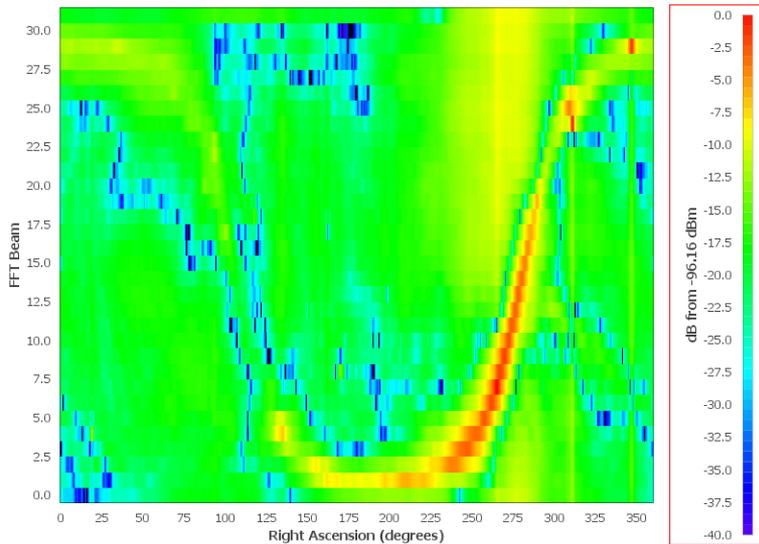


# Noiseless Pittsburgh Cylinder Visibility of a Point Source

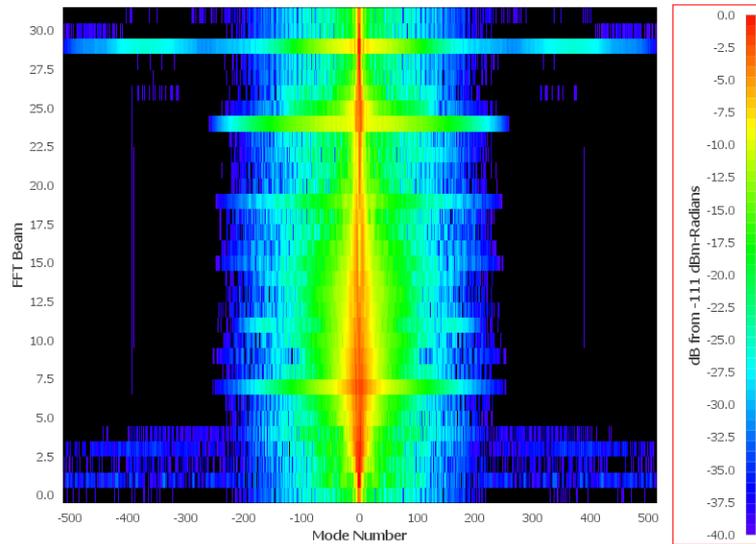


# Noiseless Pittsburgh Cylinder Visibility of Angelica Sky at 1400MHz

dB Mag Vis.

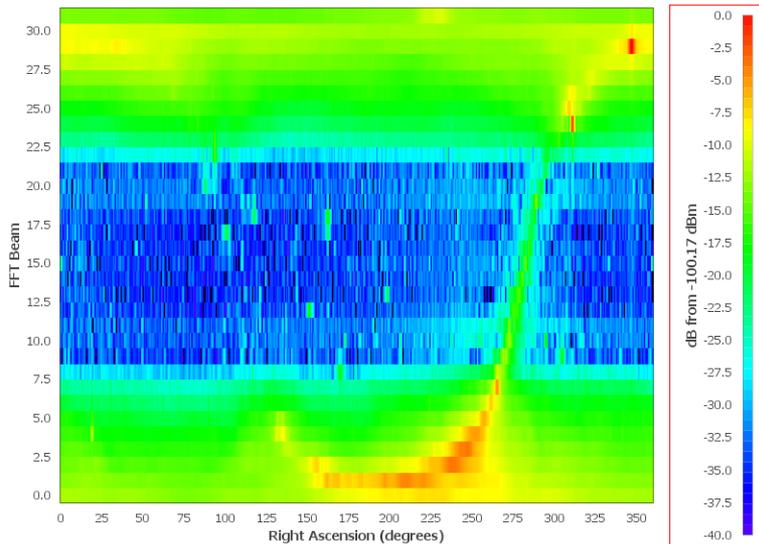


dB Vis Ra DFT

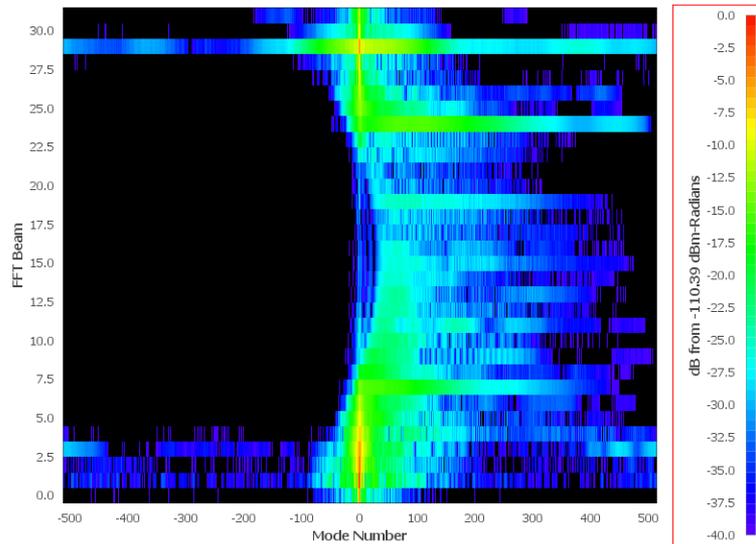


C1 x C1

dB Mag Vis.



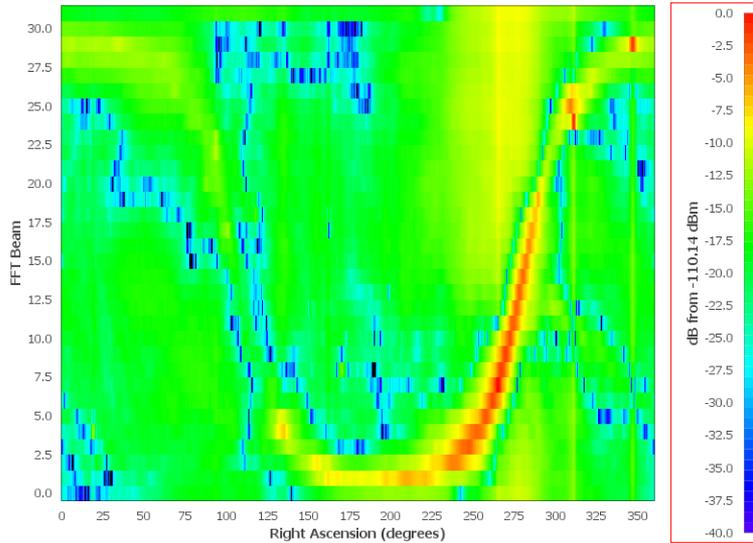
dB Vis Ra DFT



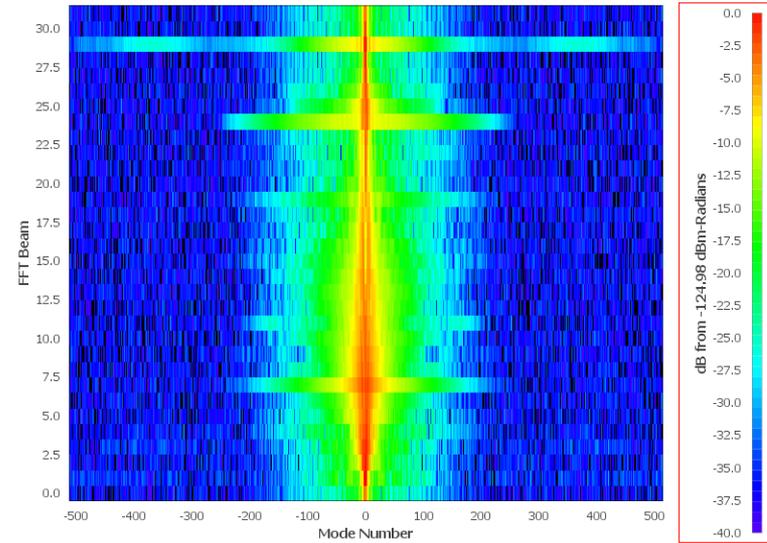
C1 x C2

# Noisy Pittsburgh Cylinder Visibility of Angelica Sky at 1400MHz

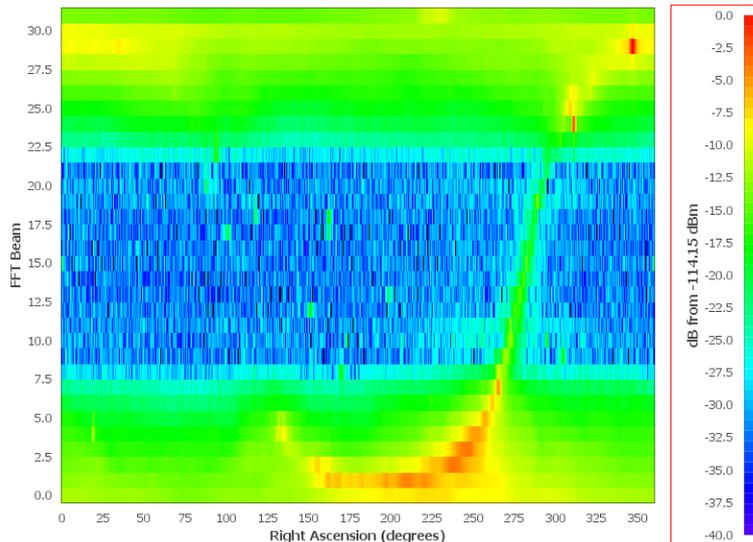
dB Mag Vis.



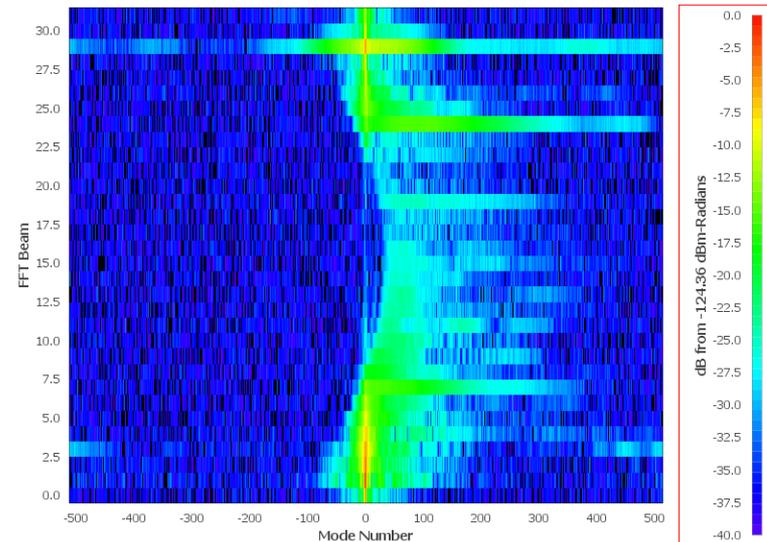
dB Vis Ra DFT



dB Mag Vis.



dB Vis Ra DFT



C1 x C1\*

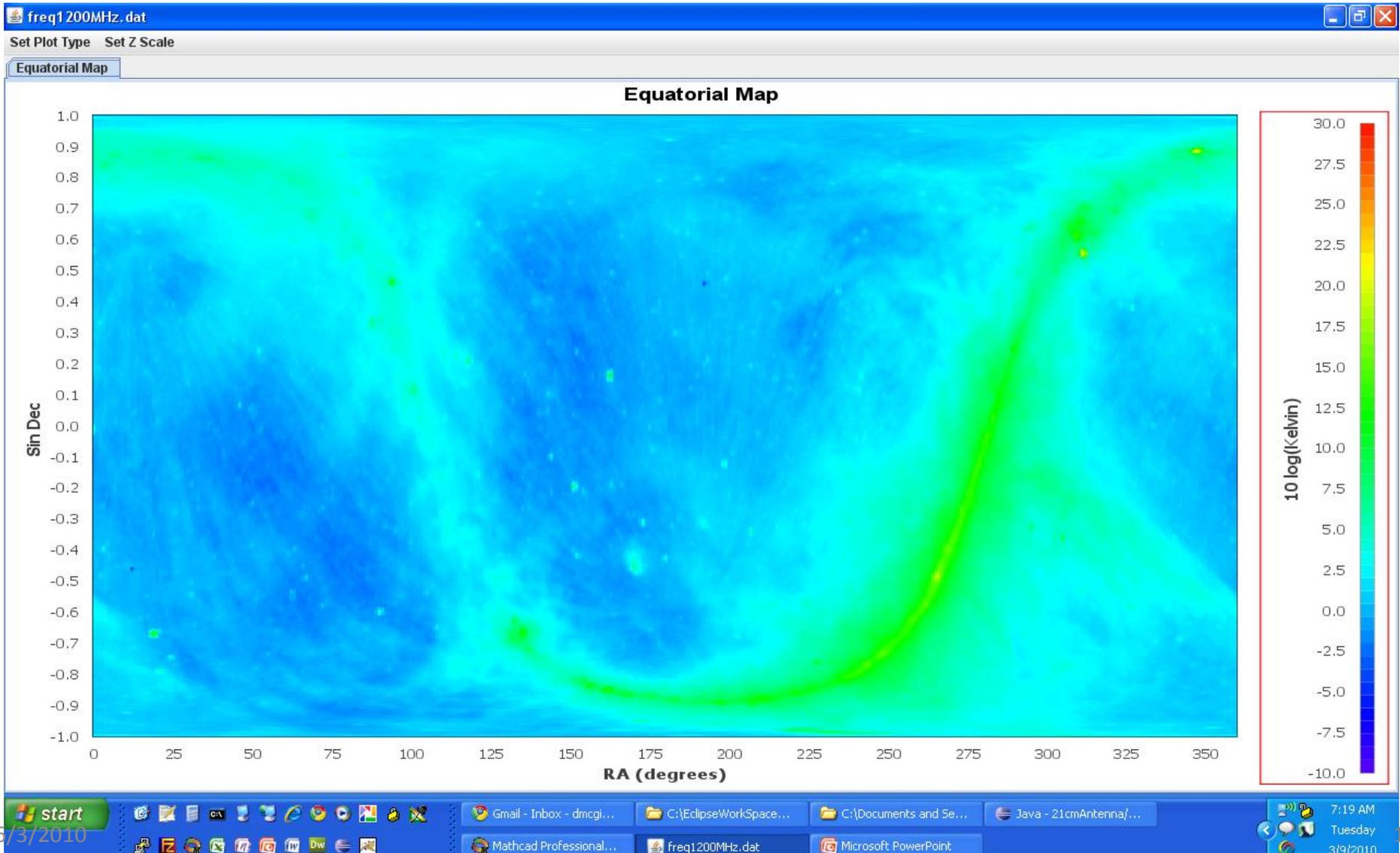
C1 x C2\*

# **FORE-GROUND SUBTRACTION FLUCTUATING SKY PATCH**

# Sky Model Subtraction Algorithm

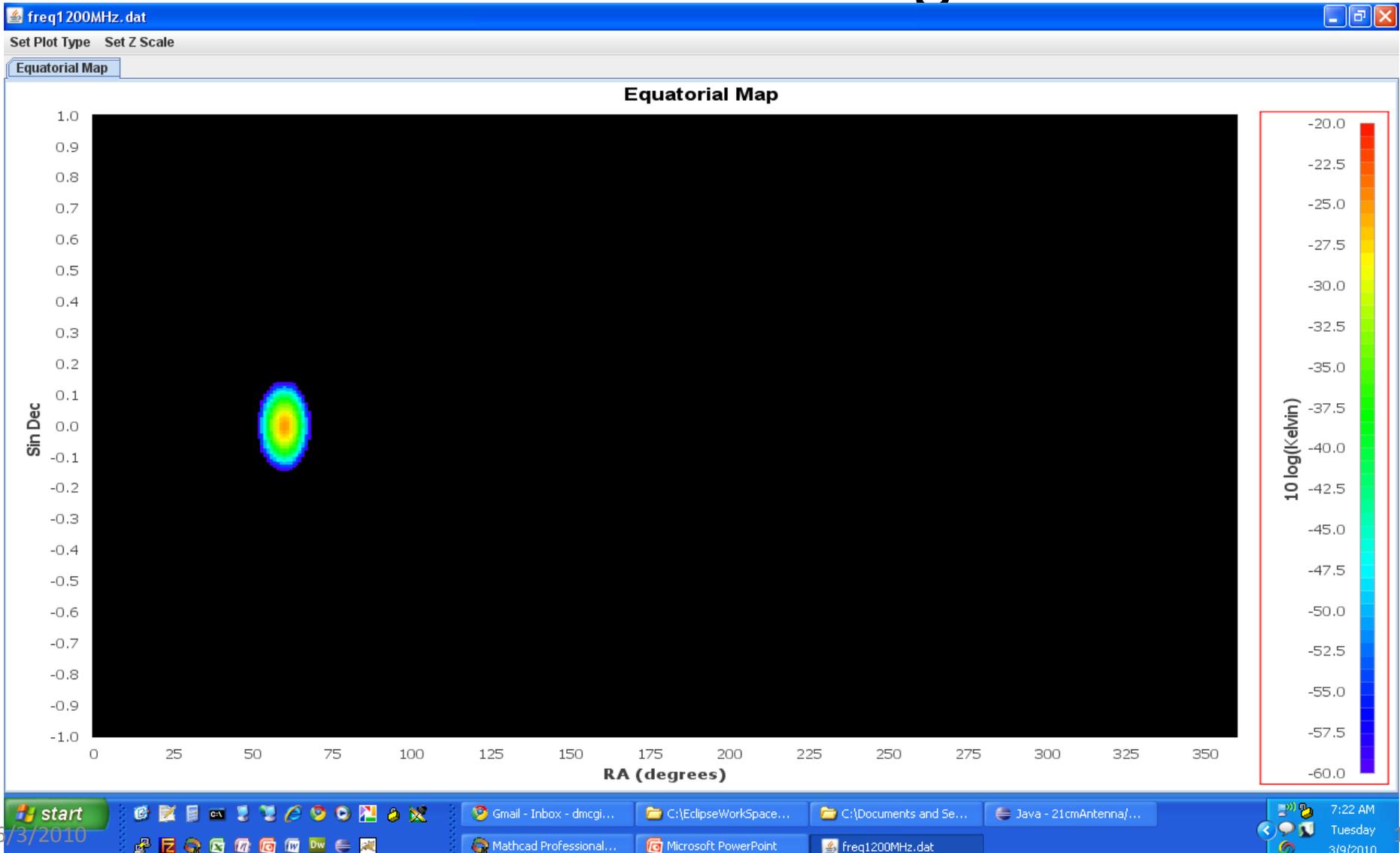
- Take cylinder visibility data and subtract a simulation of a frequency-smooth sky into a cylinder model
- From the sky difference map, fit each visibility spectrum “pixel” as a nth order polynomial in frequency
- Subtract the frequency-smoothed pixel trace from the difference map pixel by pixel
- Further FFT filter in frequency each the remaining pixel trace

# Angelica Sky Map

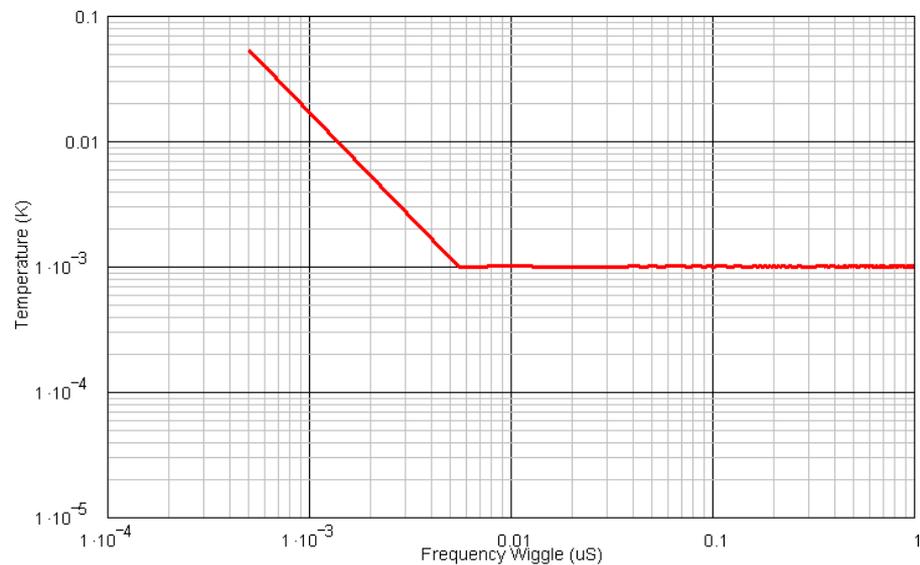
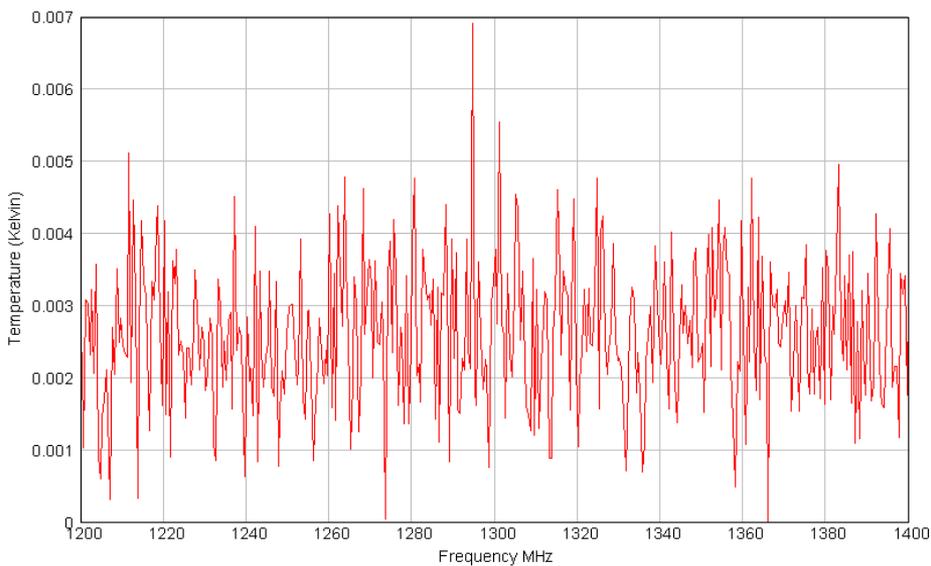


# Freq. Fluctuation Patch

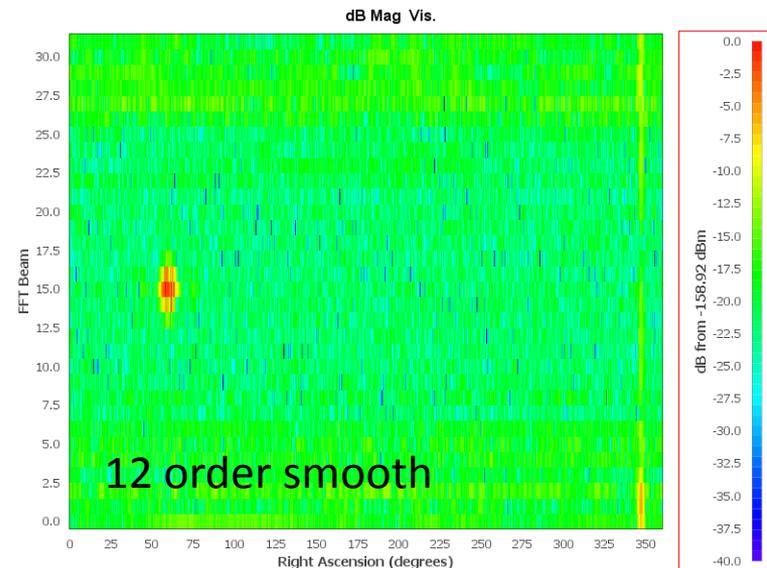
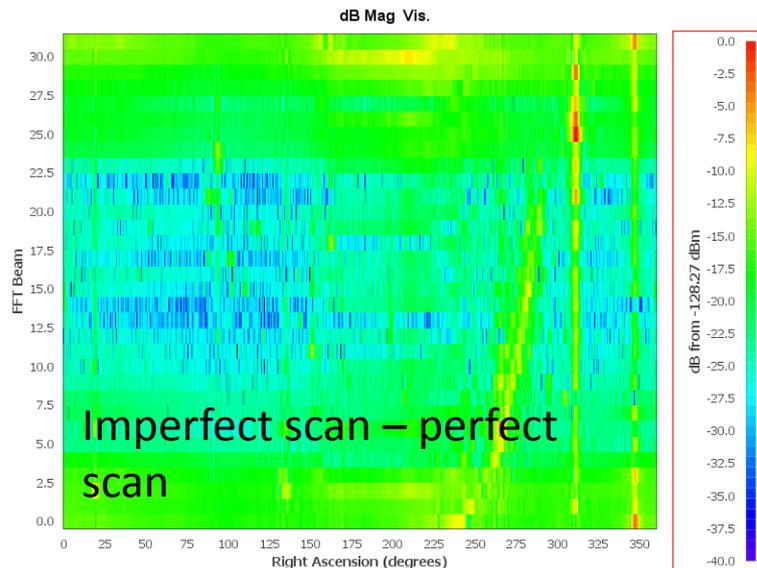
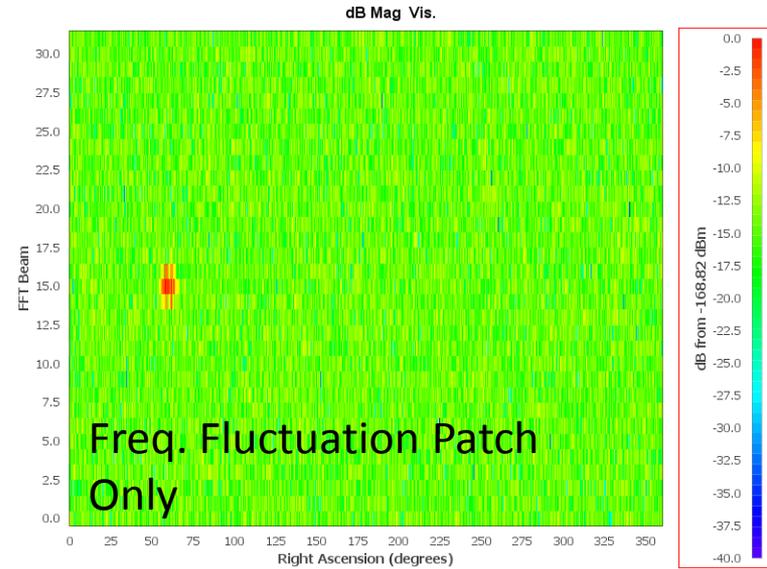
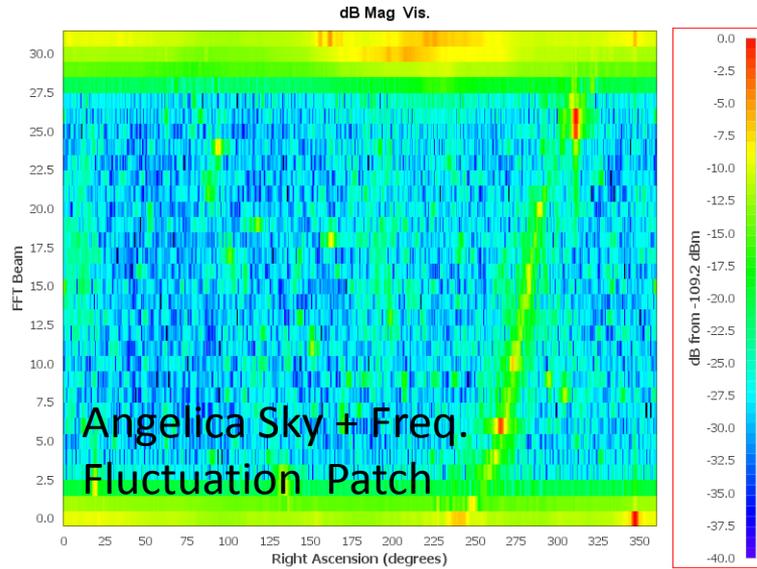
r.m.s radius = 3 degrees



# Freq. Fluctuation Patch Temperature vs Frequency

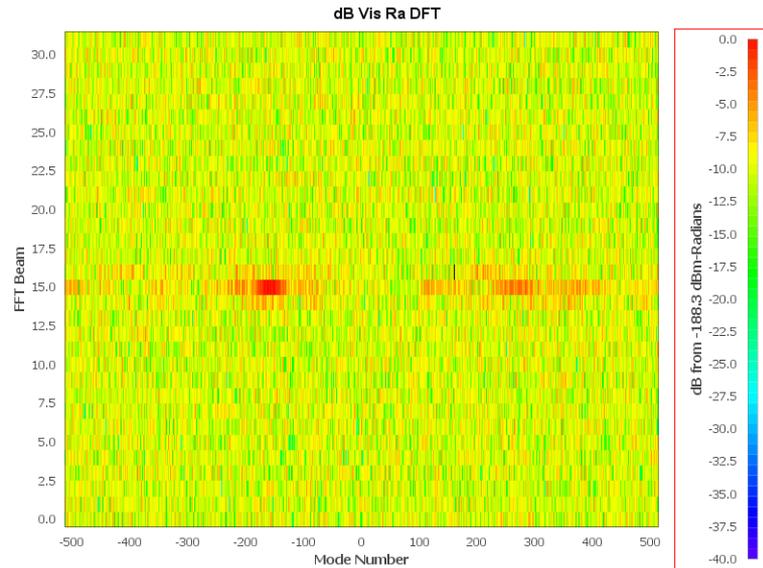
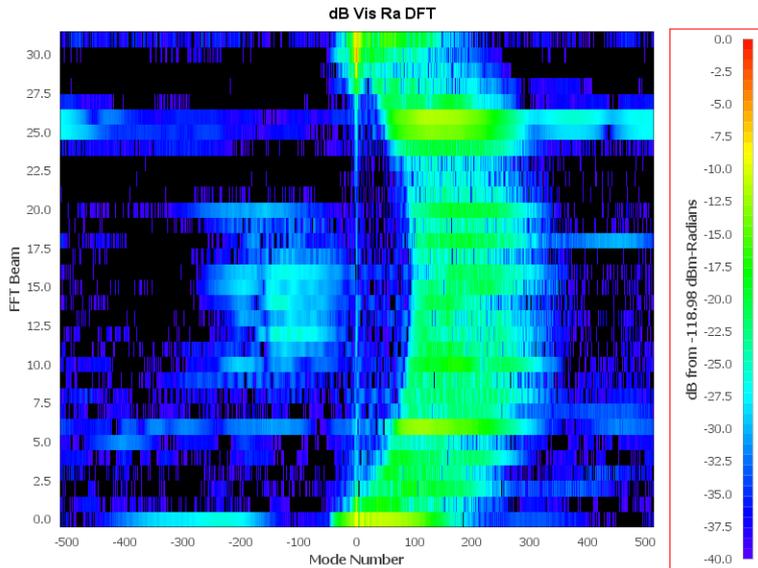


# Pittsburgh Cylinder Simulations Sky Scan



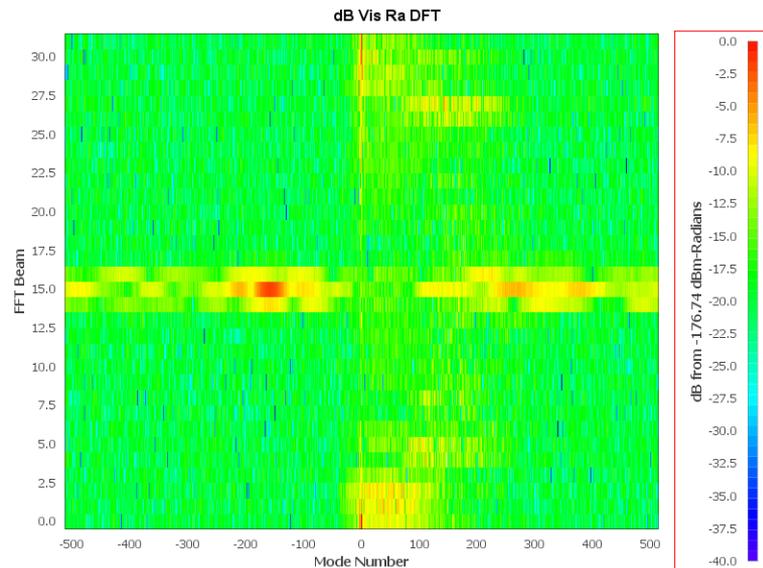
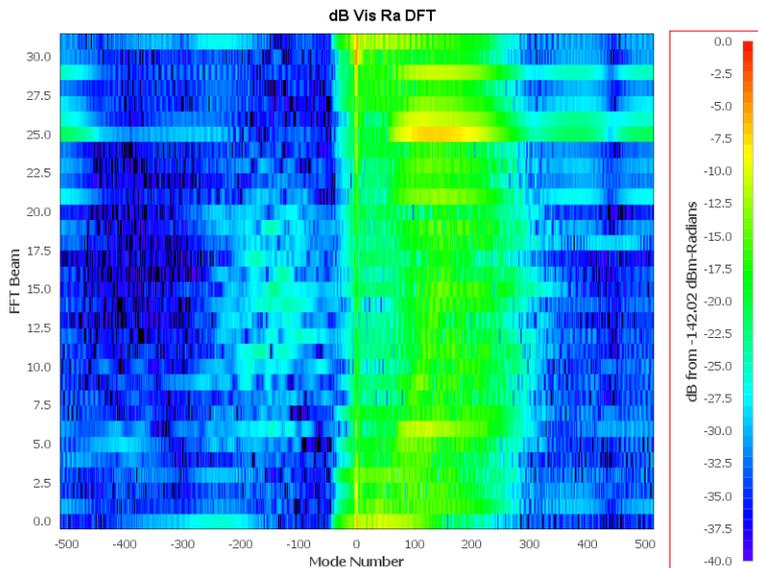
# RA DFT of Pittsburgh Cylinder Simulations

Angelica  
Sky +  
Freq.  
Fluctuat  
ion  
Patch



Freq.  
Fluctua  
tion  
Patch  
Only

Imperf  
ect  
scan –  
perfect  
scan

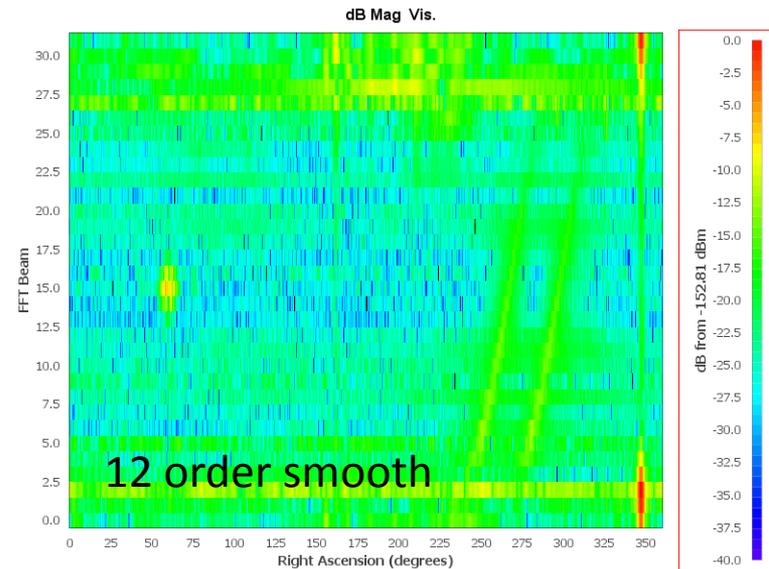
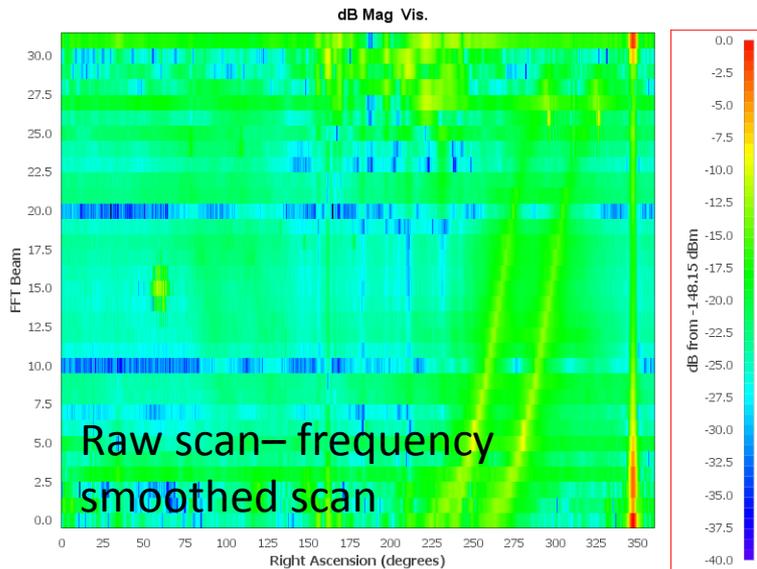
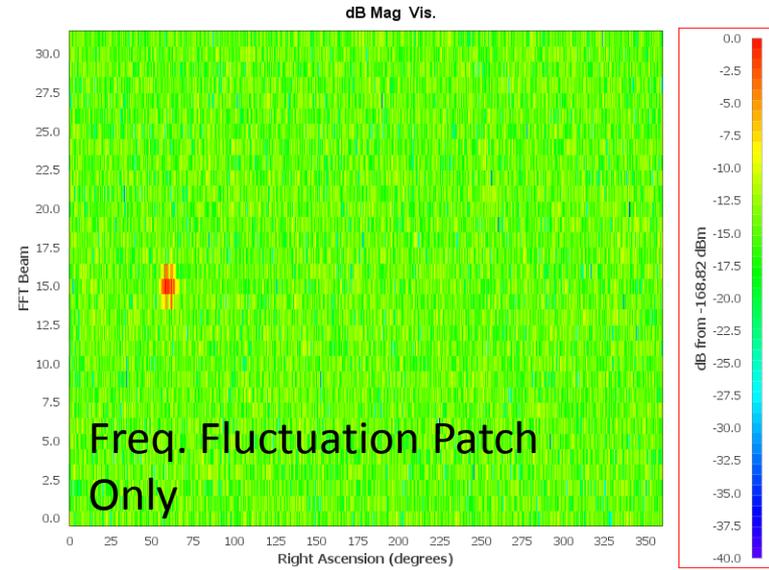
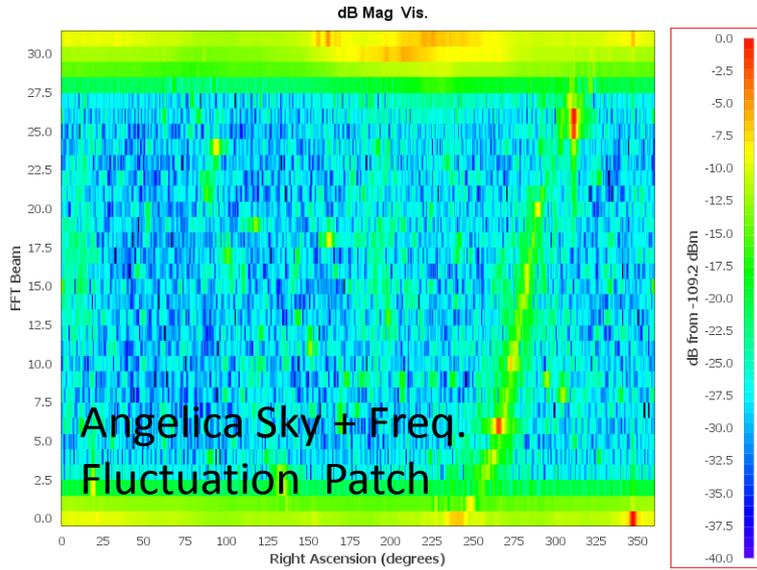


12 order  
smooth

# Frequency-Smoothed Sky Subtraction Algorithm

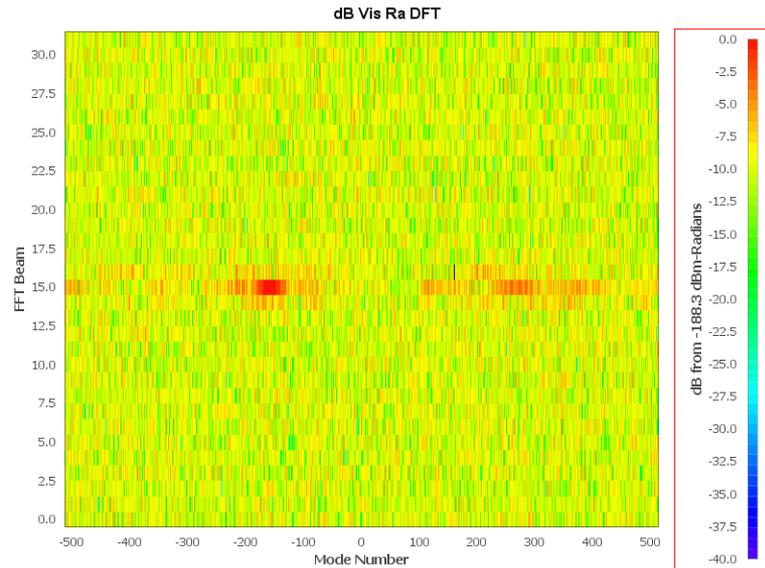
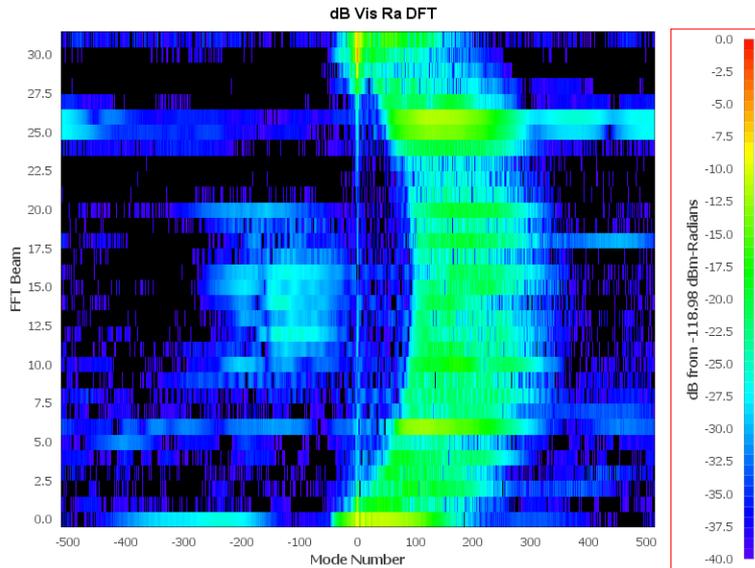
- Take cylinder visibility data smooth it along the frequency axis using a N order polynomial for each pixel
- Subtract the frequency-smoothed map from the raw map producing a difference map
- From the difference map, fit each visibility spectrum “pixel” as a nth order polynomial in frequency
- Subtract the frequency-smoothed pixel trace from the difference map pixel by pixel
- Further FFT filter in frequency each the remaining pixel trace

# Fluctuating Sky Path Simulations



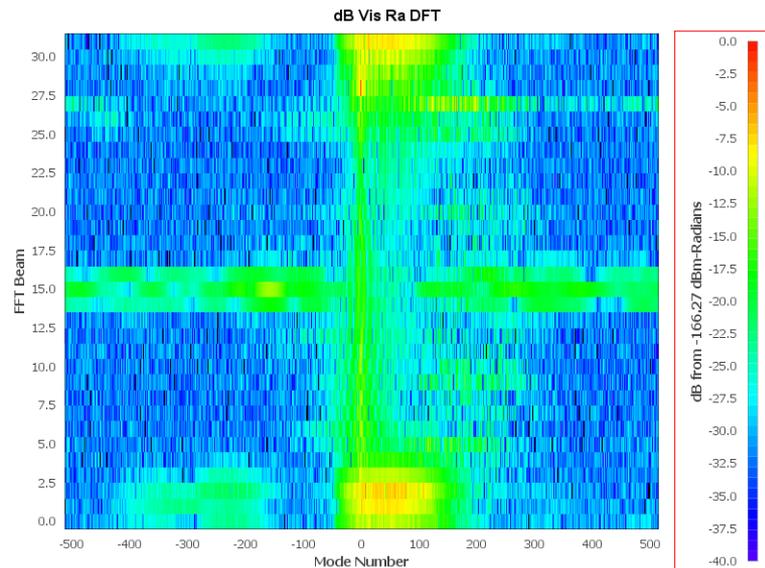
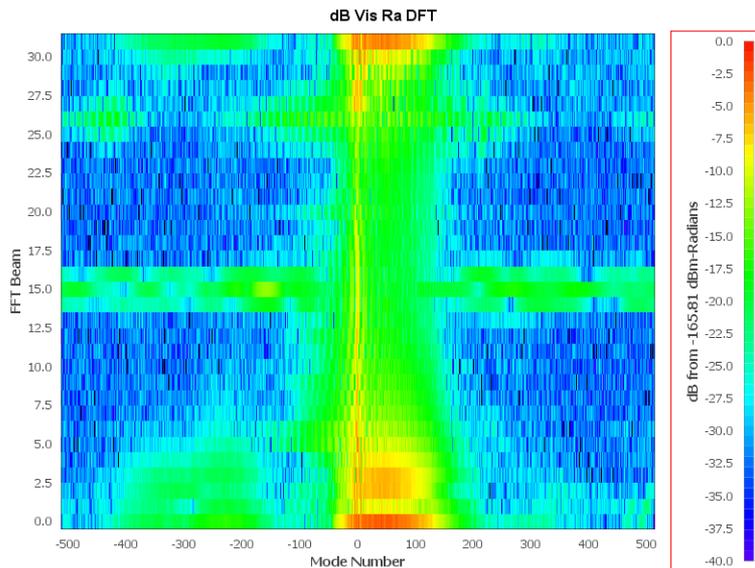
# RA DFT of Pittsburgh Cylinder Simulations

Angelica  
Sky +  
Freq.  
Fluctuat  
ion  
Patch



Freq.  
Fluctua  
tion  
Patch  
Only

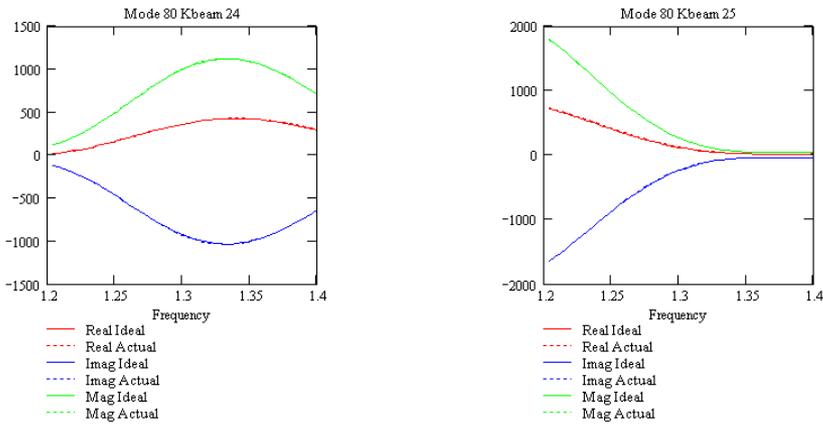
Raw  
scan –  
smooth  
ed scan



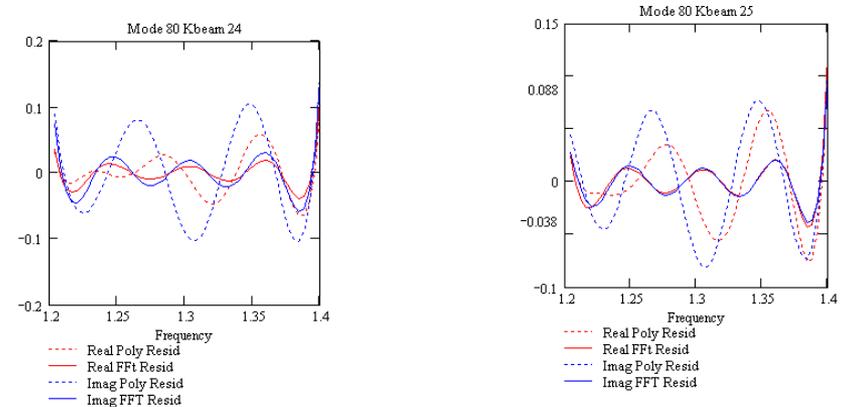
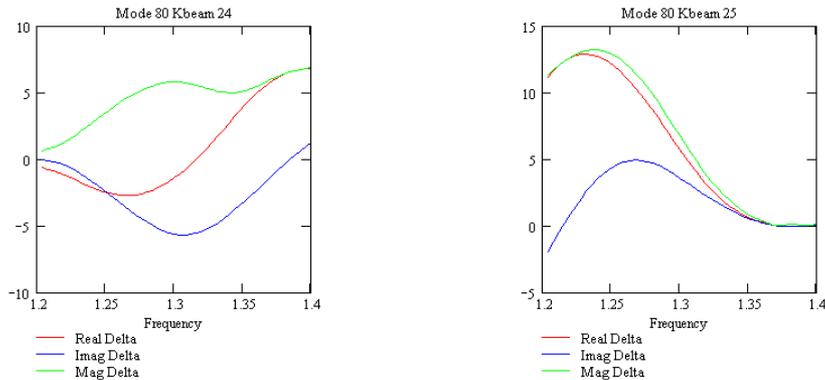
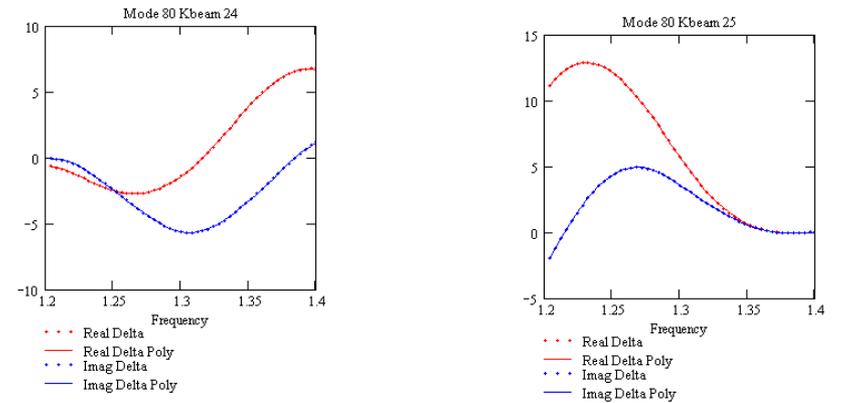
12 order  
smooth

# Mode Mixing Smoothness

“Hot Pixel” track before Sky Subtraction



“Hot Pixel” track after Sky Subtraction



# **CYLINDER VISIBILITY MODELER AND K-SPACE SKY RECONSTRUCTION**

# Cylinder Modeler

## Sky Reconstruction from Cylinder Visibilities

Dave McGinnis  
June 1, 2010

### Visibility

This note will consider the reconstruction of the sky from the measured visibilities from a pair of cylinder antenna arrays. It is assumed that the cylinders fixed and are oriented along the meridian. Each cylinder is populated with N feeds spaced uniformly along the length. The output voltage of each feed provides an input of a spatial Fourier transform along the cylinder length. The spatial Fourier transform forms N beams along the length of the cylinder.

For a pair of cylinders the visibility between cylinders is formed for each beam. As the sky drifts through the cylinder beam, the visibility for beam k is:

$$v_k(\varphi) = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta, \phi)}{\lambda^2} T(\theta, \varphi - \phi) \cos(\theta) d\theta d\phi \quad (1)$$

Where  $\varphi$  is the time of the day (in units of angle),  $\lambda$  is the wavelength and  $T$  is the power flux of the sky. The cylinder pair Fourier area is defined as

$$\tilde{A}_k(\theta, \phi) = \tilde{a}_{k,c1}(\theta, \phi) (\tilde{a}_{k,c2}(\theta, \phi))^* \quad (2)$$

where the subscripts c1, c2 indicate cylinder 1 and cylinder 2, respectively. The Fourier root area of a cylinder is defined as

$$\tilde{a}_{k,c}(\theta, \phi) = \sum_n a_n(\theta, \phi) e^{-j\beta(\theta, \phi) \cdot r_{n,c}} e^{j2\pi k \frac{r_n}{N}} \quad (3)$$

Where  $n$  is the feed number,  $r_{n,c}$  is the global location of the feed and  $\beta$  is the incoming wave vector:

$$\hat{\beta}(\theta, \phi) = \frac{2\pi}{\lambda} (\sin(\theta)\hat{x} + \cos(\theta)\sin(\phi)\hat{y}) \quad (4)$$

It is assumed that the length of the cylinders is in the x direction.

### Sky Expansion

Since the sky is periodic, it can be expanded in a Fourier series:

$$T_c(\theta, \phi) = \sum_l \sum_m \hat{T}_{m,l} e^{jl\pi \sin(\theta)} e^{jm\phi} \quad (5)$$

Since  $T_c$  is a complex function but the sky temperature must be a real function, the sky temperature can be written as:

$$T(\theta, \phi) = T_c(\theta, \phi) + (T_c(\theta, \phi))^* \quad (6)$$

$$T(\theta, \phi) = \sum_l \sum_m \hat{T}_{m,l} e^{jl\pi \sin(\theta)} e^{jm\phi} + \sum_l \sum_m (\hat{T}_{m,l})^* e^{-jl\pi \sin(\theta)} e^{-jm\phi} \quad (7)$$

$$T(\theta, \phi) = \sum_l \sum_m (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*) e^{jl\pi \sin(\theta)} e^{jm\phi} \quad (8)$$

Substituting Equation 8 into Equation 1,

$$v_k(\varphi) = \sum_l \sum_m (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*) e^{jm\varphi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta, \phi)}{\lambda^2} e^{jl\pi \sin(\theta)} e^{-jm\phi} \cos(\theta) d\theta d\phi \quad (9)$$

### Fourier Transform of Cylinder Visibilities

Now assume that the visibility is measured at N discrete times during the day. The visibility will be periodic with a period of a day so we can expand the visibility into a Fourier series.

$$v_k(\varphi_n) = \sum_{m'} \tilde{V}_{k,m'} e^{jm'\varphi_n} \quad (10)$$

where

$$\tilde{V}_{k,m'} = \frac{1}{N} \sum_n v_k(\varphi_n) e^{-jm'\varphi_n} \quad (11)$$

Substituting Equations 9 into Equations 11,

$$\tilde{V}_{k,m} = \sum_l \tilde{A}_{k,l,m} (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*) \quad (12)$$

where:

$$\tilde{A}_{k,l,m} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta, \phi)}{\lambda^2} e^{jl\pi \sin(\theta)} e^{-jm\phi} \cos(\theta) d\theta d\phi \quad (13)$$

The discrete approximation for Equation 13 is:

$$\tilde{A}_{k,l,m} \approx \frac{4\pi}{N_p N_q} \sum_{q=0}^{N_q} \sum_{p=0}^{N_p} \frac{\tilde{A}_k(\theta_p, \phi_q)}{\lambda^2} e^{jl(\pi \sin(\theta))_p} e^{-jm\phi_q} \quad (14)$$

The matrix form of Equation 13 is:

$$[\tilde{V}_m] = [\tilde{A}_m][\tilde{T}_m] \quad (15)$$

### Multiple Visibilities

There can be a number of combination of cylinder visibilities that have a non-zero component a given right ascension mode m. However, the sky mode temperature at mode m must be unique.

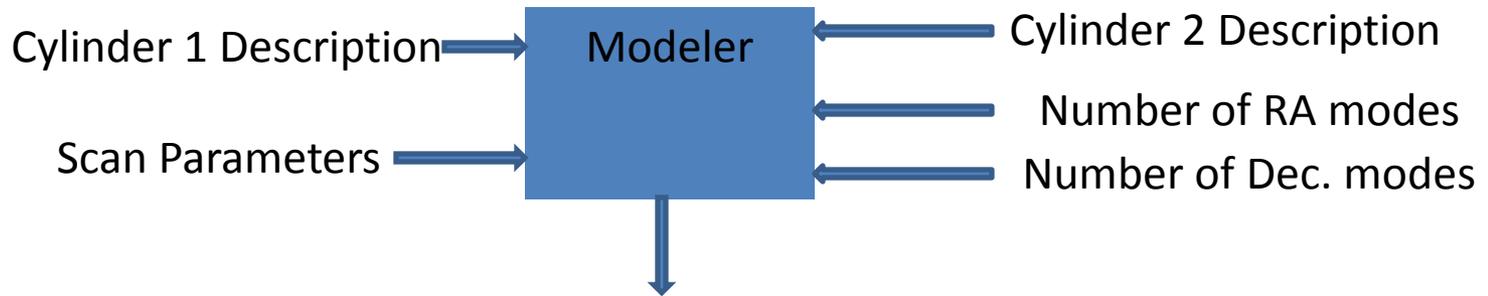
$$[\tilde{V}_m]^{(1)} = [\tilde{A}_m]^{(1)} [\tilde{T}_m] \quad (16)$$

$$[\tilde{V}_m]^{(2)} = [\tilde{A}_m]^{(2)} [\tilde{T}_m] \quad (17)$$

To satisfy both equations in a least squares fit, equations 16 and 17 can be combined:

$$\begin{aligned} & \left\{ \left\{ [\tilde{A}_m]^{(1)} \right\}^T \right\}^* [\tilde{V}_m]^{(1)} + \left\{ \left\{ [\tilde{A}_m]^{(2)} \right\}^T \right\}^* [\tilde{V}_m]^{(2)} \\ & = \left( \left\{ \left\{ [\tilde{A}_m]^{(1)} \right\}^T \right\}^* + \left\{ \left\{ [\tilde{A}_m]^{(2)} \right\}^T \right\}^* \right) [\tilde{T}_m] \end{aligned} \quad (18)$$

# Cylinder Visibility Modeler



Model Mode Matrix

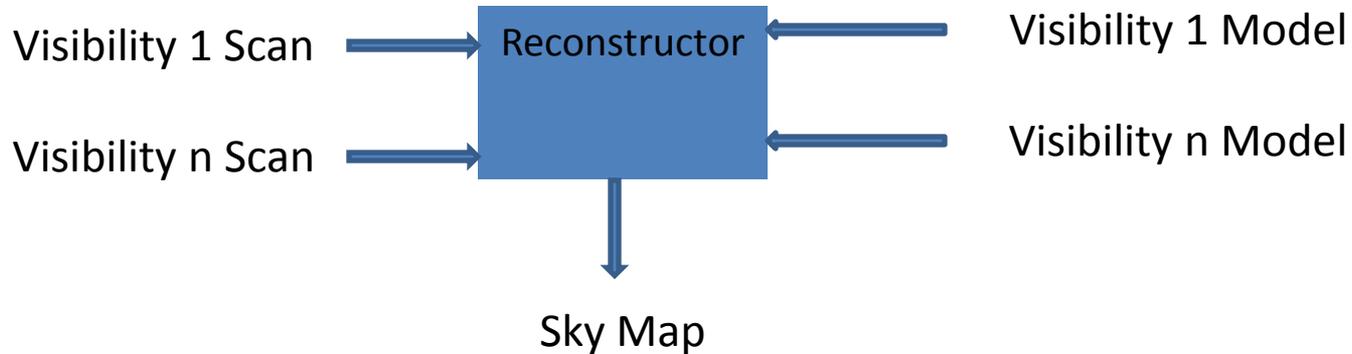
$$\tilde{V}_{k,m'} = \frac{1}{N} \sum_n v_k(\varphi_n) e^{-jm\varphi_n}$$

$$T(\theta, \phi) = \sum_l \sum_m (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*) e^{jl\pi \sin(\theta)} e^{jm\phi}$$

$$\hat{A}_{k,l,m} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta, \phi)}{\lambda^2} e^{jl\pi \sin(\theta)} e^{-jm\phi} \cos(\theta) d\theta d\phi$$

$$\tilde{V}_{k,m} = \sum_l \hat{A}_{k,l,m} (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*)$$

# Sky Reconstruction



$$[\tilde{V}_m]^{(1)} = [\hat{A}_m]^{(1)} [\tilde{T}_m]$$

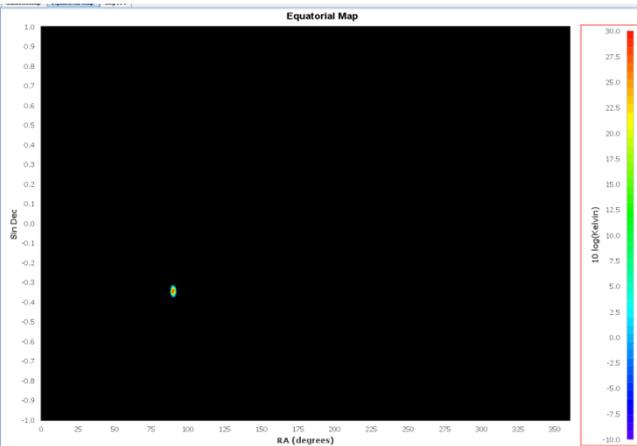
$$[\tilde{V}_m]^{(2)} = [\hat{A}_m]^{(2)} [\tilde{T}_m]$$

$$\begin{aligned} & \left\{ \left\{ [\hat{A}_m]^{(1)} \right\}^T \right\}^* [\tilde{V}_m]^{(1)} + \left\{ \left\{ [\hat{A}_m]^{(2)} \right\}^T \right\}^* [\tilde{V}_m]^{(2)} \\ &= \left( \left( \left\{ \left\{ [\hat{A}_m]^{(1)} \right\}^T \right\}^* + \left\{ \left\{ [\hat{A}_m]^{(2)} \right\}^T \right\}^* \right) [\tilde{T}_m] \end{aligned}$$

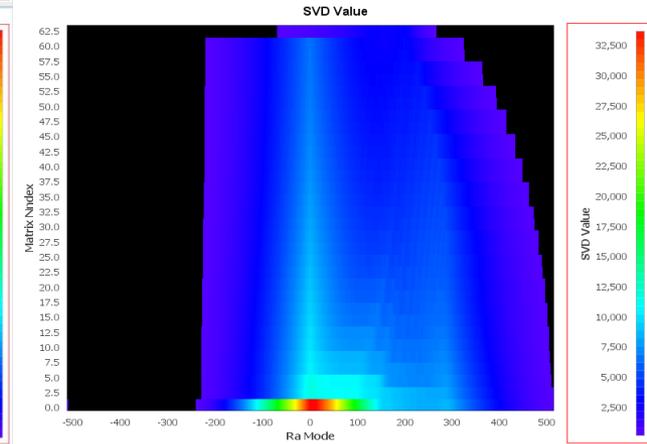
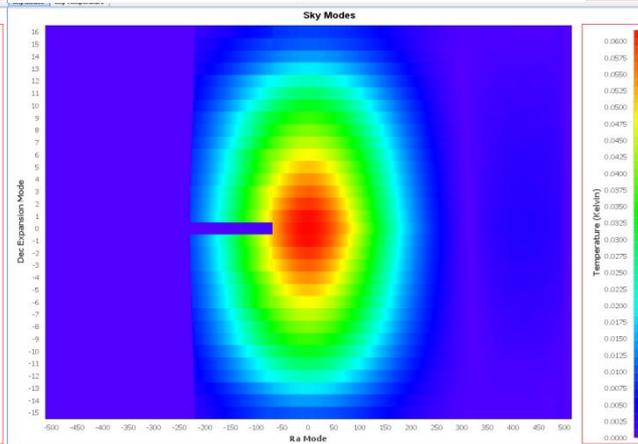
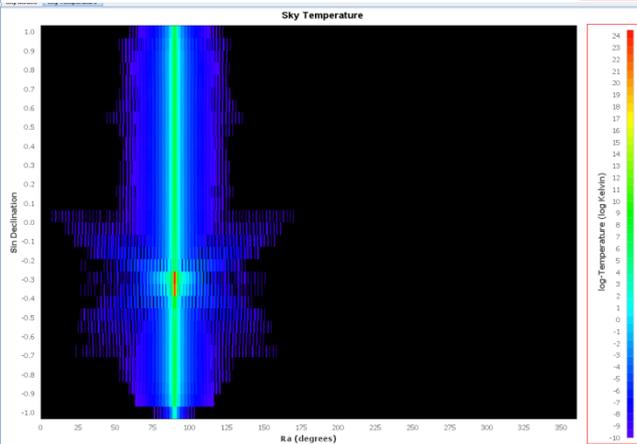
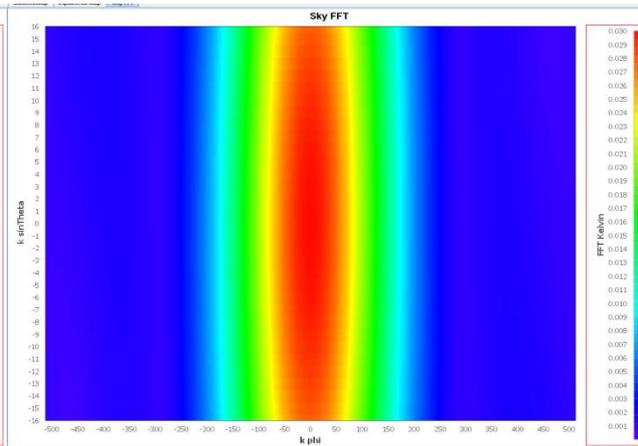
# Point Source Sky Map Reconstruction

(Pittsburgh Cylinders)

Input Sky



Input K-Space



Output Sky

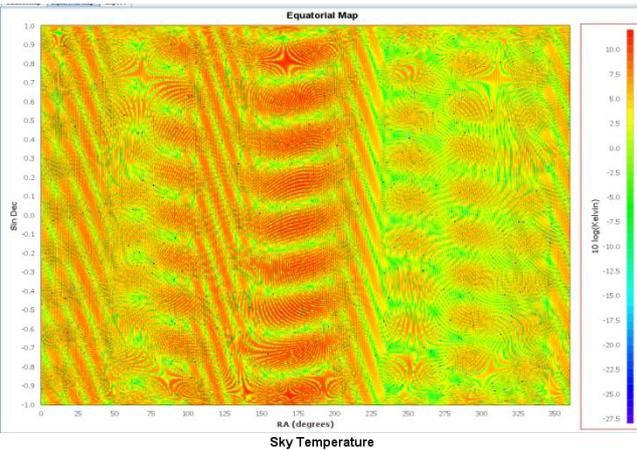
Output K-Space

SVD Values

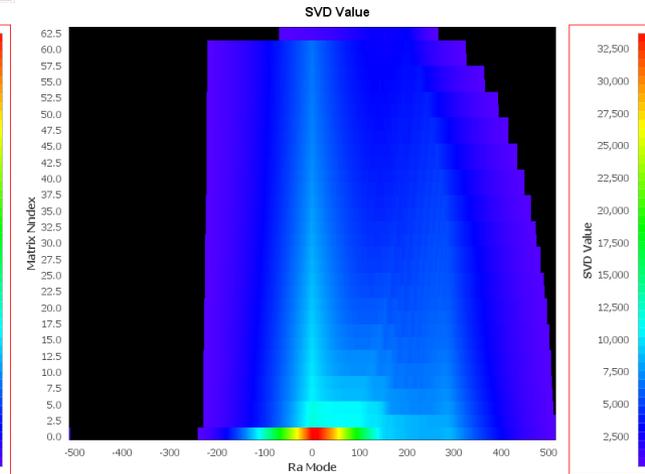
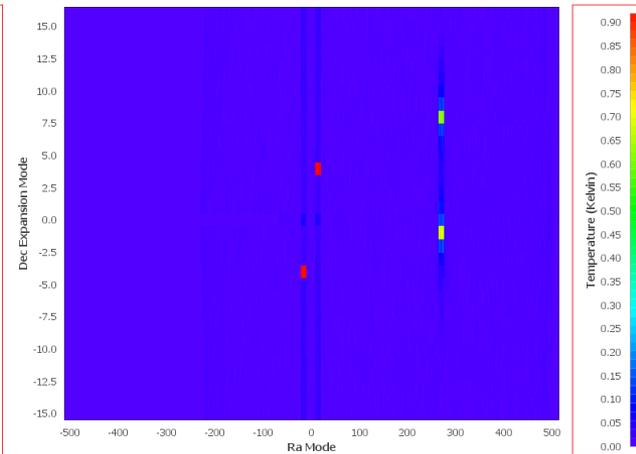
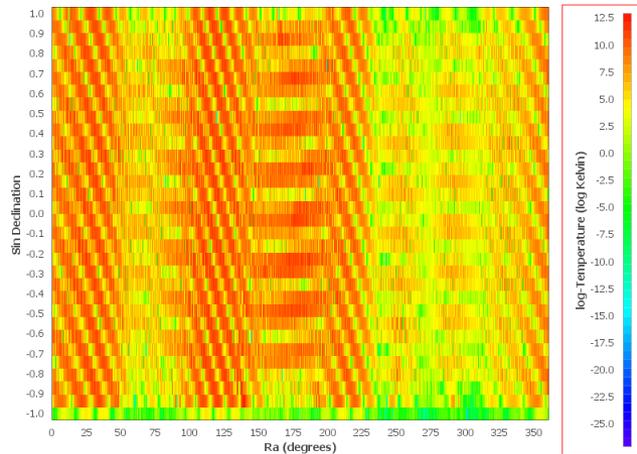
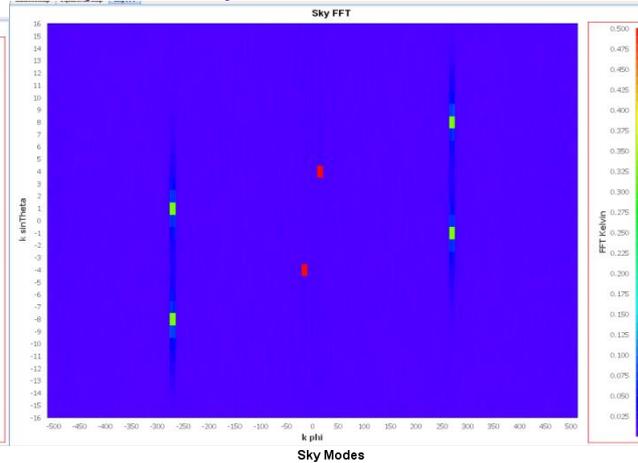
# K-Wave Sky Map Reconstruction

(Pittsburgh Cylinders)

Input Sky



Input K-Space



Output Sky

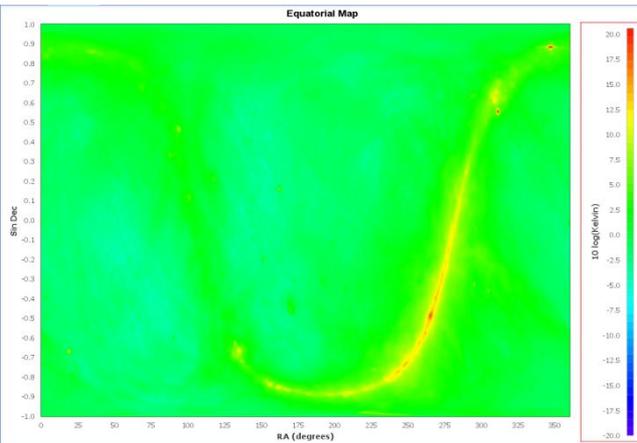
Output K-Space

SVD Values

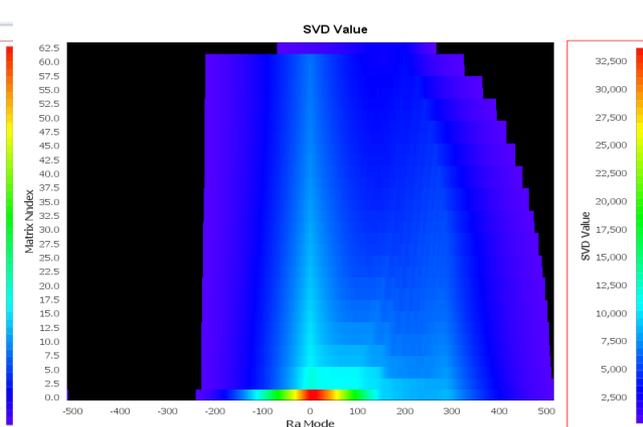
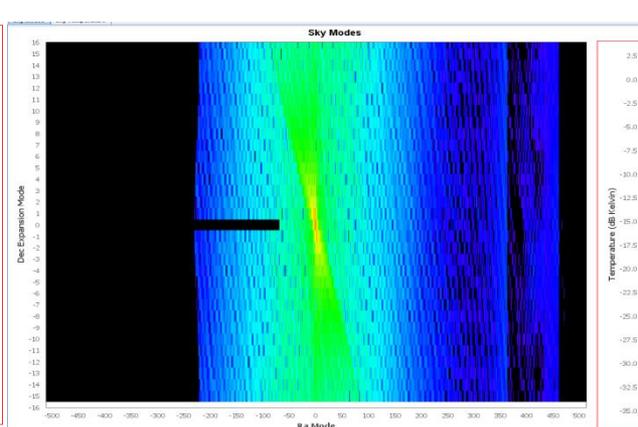
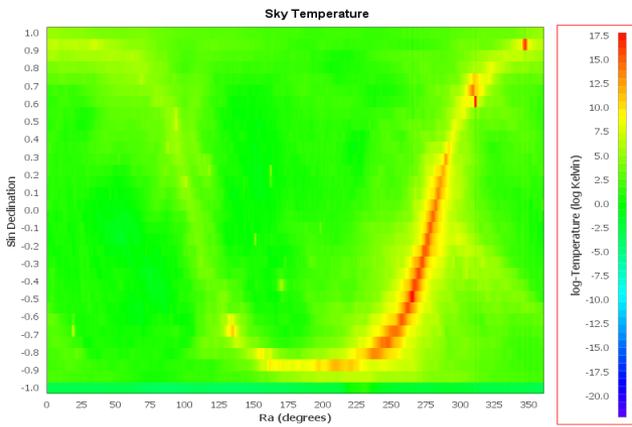
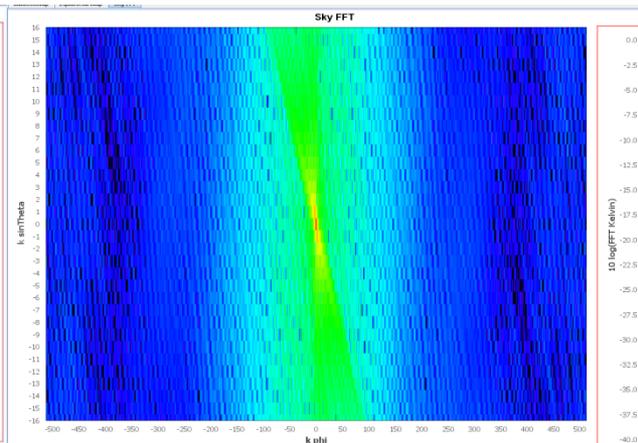
# Angelica Sky Map Reconstruction

(Pittsburgh Cylinders)

Input Sky



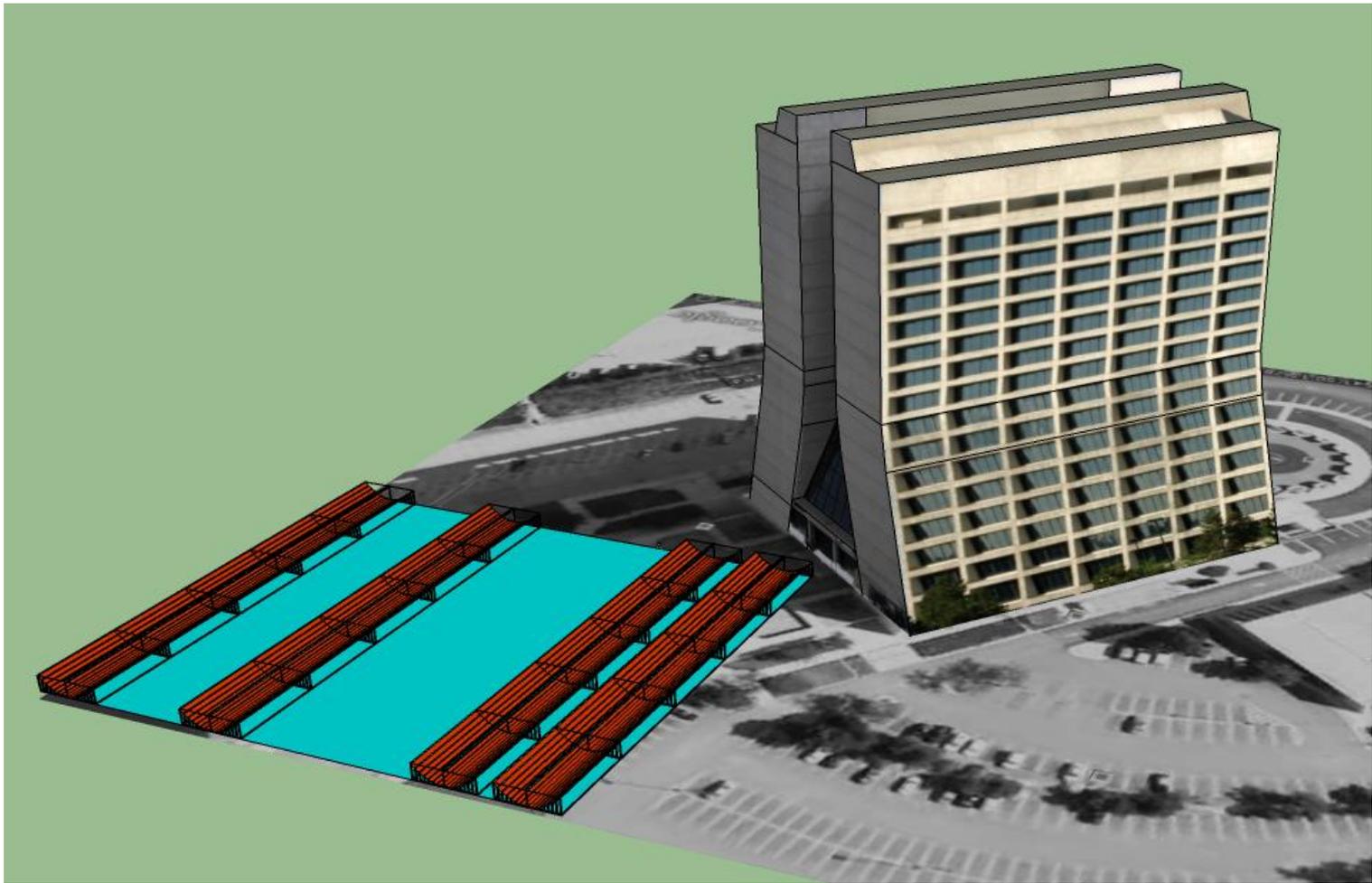
Input K-Space



Output Sky

Output K-Space

SVD Values



## **NEXT STEP – FULL SCALE TELESCOPE SIMULATION AND RECONSTRUCTION**

# Foreground Removal using BAO Simulations

- We need  $\sim 128$  feeds/cylinder to resolve the first BAO peak
- The simulation size increases  $\sim$  cubically with the number of feeds
  - 256 feeds and 4 cylinders requires  $\sim 500$  times more computing than 32 feeds and 2 cylinders
  - In addition, the simulation has to be done over a number of frequency slices ( $>100$  slices)
    - To simulate a telescope with 256 feeds/cylinder and 4 cylinders (7 cylinder combinations) at 100 frequency slices requires 1.8 years of cpu time.
    - To reconstruct the telescope, , the telescope model matrices requires a total of 1.500 Terabytes of disk space.

# FNAL-KICP Joint Cluster

- Since the last review, we have moved the simulation software to the FNAL-KICP cluster which has >1200 cpu's available.
  - Using 112 cpu's, it takes about 140 hours to simulate the 256 feed telescope.
    - This has been done.
  - We are currently having issues with the large storage space required for the telescope model matrices.
    - Space is not an issue. There is 97 Terabytes available with 12 Terabytes available to a single user.
    - Accessing the data rapidly is the issue. – it can be solved.

# FNAL-KICP Joint Cluster



Fermi National Accelerator Laboratory

## FNAL - KICP Joint Cluster

### Hardware Details

Component	Properties	Total
Master node	8-way, 16GB, fulla	2
Quad-socket dual-core nodes	8-way, 128GB, cc001	1
Quad-socket dual-core nodes	8-way, 32GB, cc002-004	3
Dual-socket quad-core nodes	8-way, 16GB, cc005-...	149
Total # of cores		1240
Infiniband nodes		24

```
Usage Totals: 838/1224 Procs, 123/153 Nodes, 120/180 Jobs Running
Node States: 2 down,offline 50 free 92 job-exclusive 9 offline

visible CPUs: 0,1,2,3,4,5,6,7
1 2 3 4 5 6 7 8 9 0
-----
cc001 [M][M][M][M] zzzzzzzz ..... [P]..... [C]..... [B]t..... dT..... [L]..... [X].....
cc011 vvv v [O][O][g].. j1111111 MMMIIMI [P][P][P][P][P] [M][M][M][M][M][M] [C]i..... [K]..... ***** [2].....
cc021 p[.H]..... AAAAAAAA zU..... fgggggg. Tgggggg. [R]..... g[ggggg]..... [L]..... [V]..... CCCCCC.
cc031 yyyyyyyy [N]..... 00000000 ..... [C]CCCCC. ***** e..... rrrrrrrr A....b. [G].....
cc041 [I]..... AAAAAAAA C1CCCC. yyyyyyyy yyyyyyyy [V]..... y[lyyy].. [uuuuuu] HHHHHHHH [J].....
cc051 [E][ff]... dddddddd HHHHHHHH ..... [uuuuuu]..... [pppp] iiii1111 dddddddd iiii1111 j1111111
cc061 BBBBLLL jkkkk [uu] ..... ssssssss j111111 P[PPPPPP] P[PPPPPP] rrrrrrrr xxxxxxxx *****
cc071 AAAARRRR SSSS [uu] NNNNaaaa xxxxxxxx ..... DDDDDDDD DDDDDDDD JJJJJJJJ cccccccc *****
cc081 [uuuuuu] EEEEEEEE ..... ***** nnnnnnnn wwwwwwv [KKKKKKK] *****
cc091 ***** ***** ***** [PPPPPP] P[PPPPPP] ..... ***** FFFFFFFF hhhhhhhh hhhhhhhh FFFFFFFF .....
cc101 ***** ***** ***** nnnnnnnn ***** ***** FFFFFFFF hhhhhhhh hhhhhhhh FFFFFFFF .....
cc111 [ttt] FFFFFFFF GGGGcccc vvvvvvvv ..... l111111 ..... *****
cc121 [KKKKKKK] ..... [u]ooUUU [SSSSSS] [SSSSSS] ssssssss bbbbbbbb ***** ***** [GGGGGGG]
cci002 [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG]
cci012 [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG]
cci022 [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG] [GGGGGGG]
```

- Quad-socket nodes contain 2.2GHz AMD Opteron 8214 processors; they are good for shared memory parallel jobs or for jobs that require very large amount of memory per node.
- Dual-socket nodes contain 2.0GHz AMD Opetron 2350 processors; they are suitable for all kinds of jobs (serial, embarrassingly parallel, shared or distributed memory). These nodes are available under the PBS queue normal.

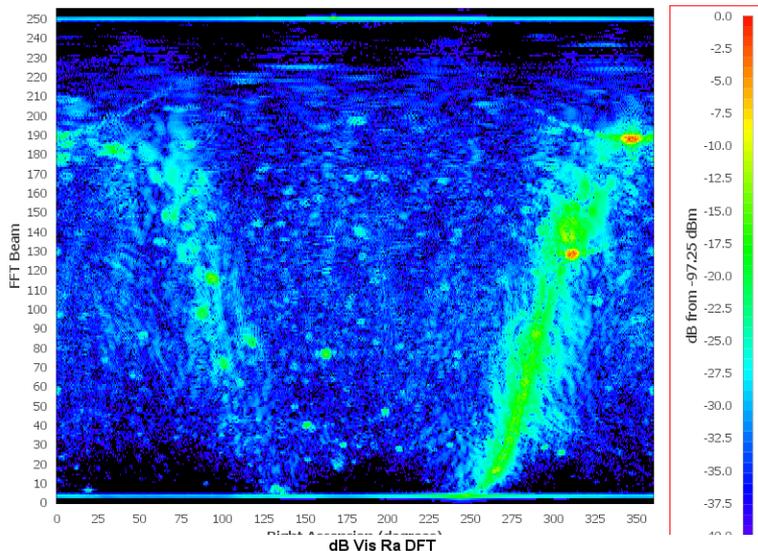
### Operating System

Scientific Linux, 64 bit

# C1-C7 "Visibility" for 256 feeds

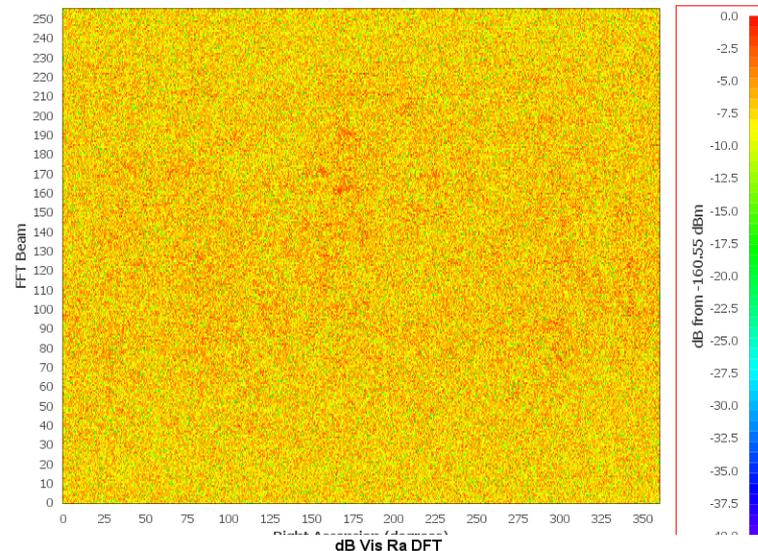
BAO 1<sup>st</sup> Peak + Angelica

dB Mag Vis.



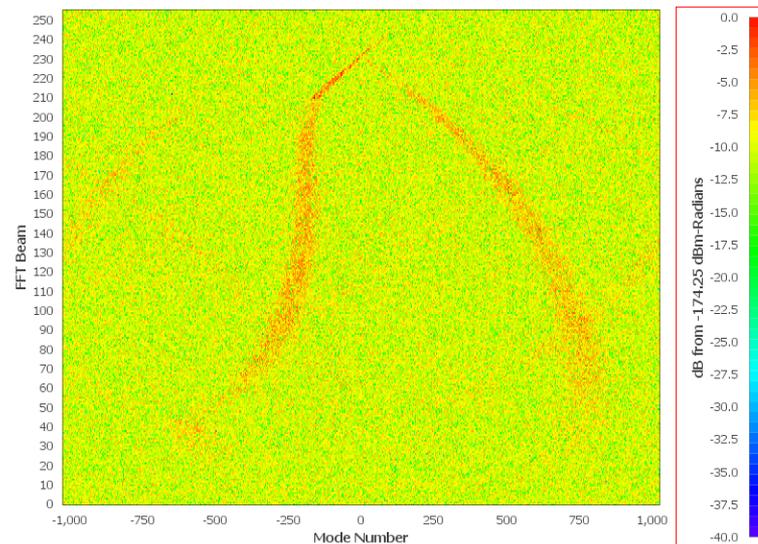
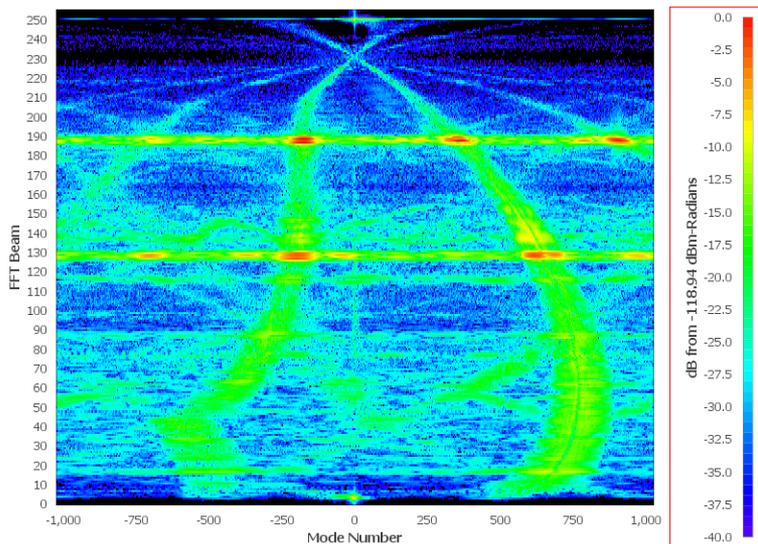
BAO 1<sup>st</sup> Peak

dB Mag Vis.



Ra Scan

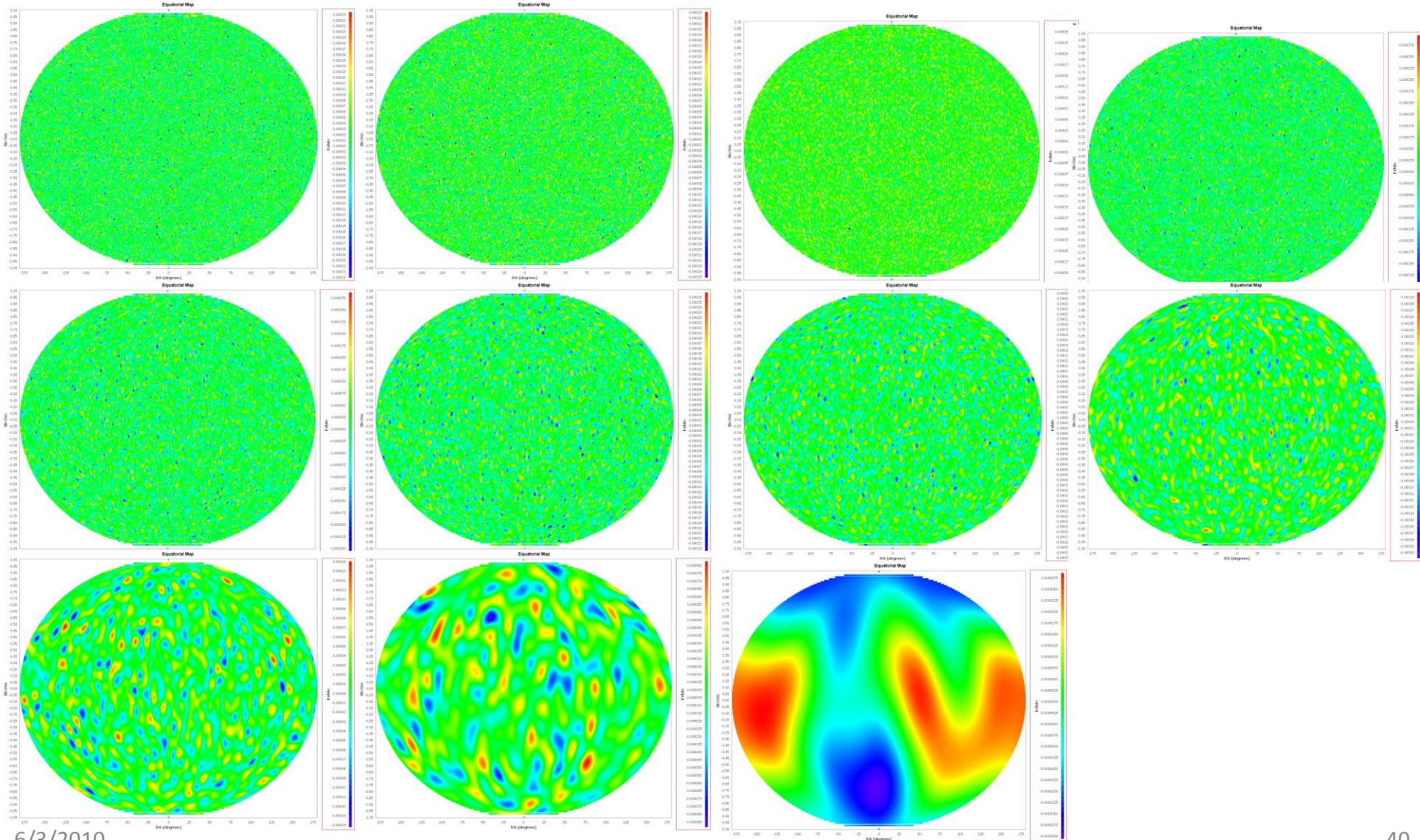
Ra FFT



# Intermediate Step to Foreground Removal

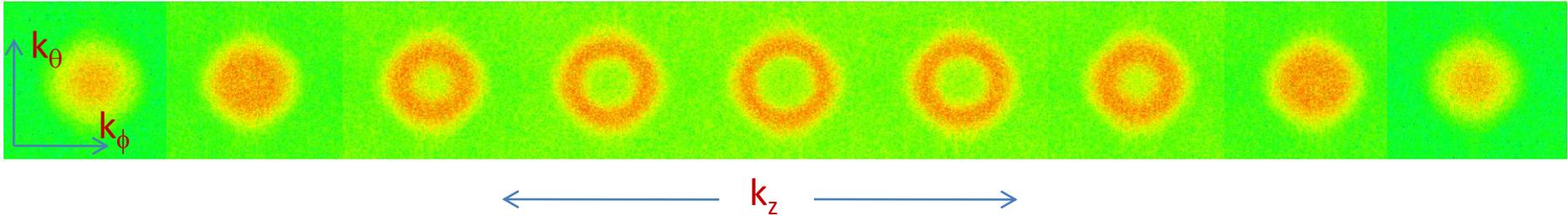
- Separate issues of telescope modeling and sky reconstruction from foreground removal algorithms.
  - Assume that the sky reconstruction will remove issues with mode mixing.
- Use sky maps with frequency-smooth foregrounds overlaid on top of BAO signal to test foreground removal algorithms
- Use BAO simulations of the first peak from Nick Gnedin
  - 1000 frequency points from 400-1400MHz
  - $N_{\text{side}}=128$

# BAO Signal First Peak from 400-1400MHz

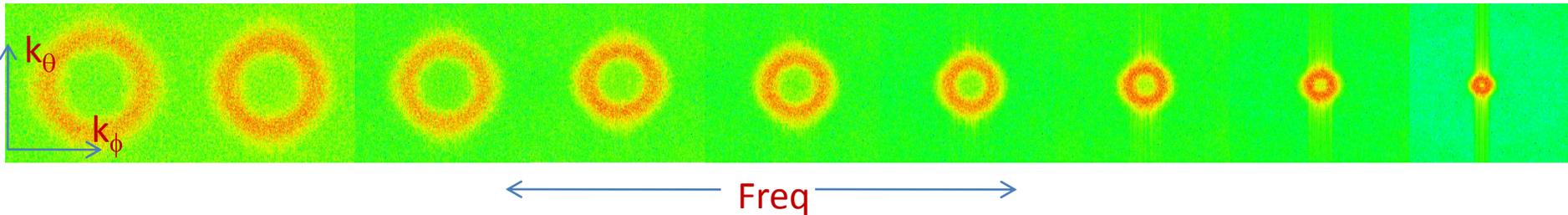


# BAO First Peak 3-D K space

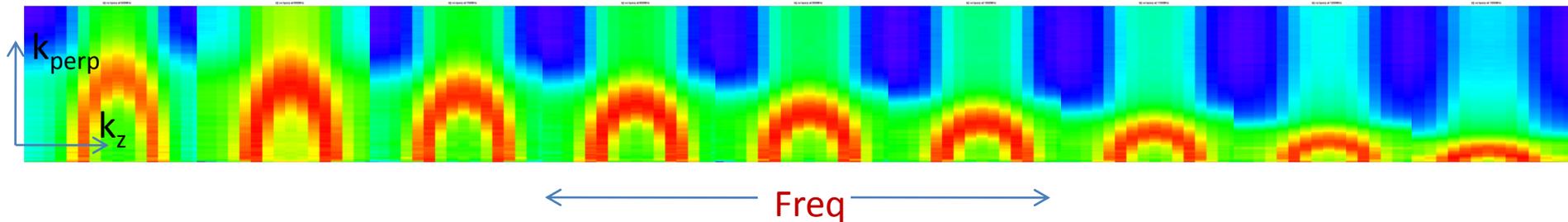
BAO First Peak in 3-D k-Space at 750 MHz – ResBw = 1/128 MHz



BAO First Peak from 500-1300MHz; Kperp at “ $k_{||} = 0$ ”; ResBW = 1/128 MHz

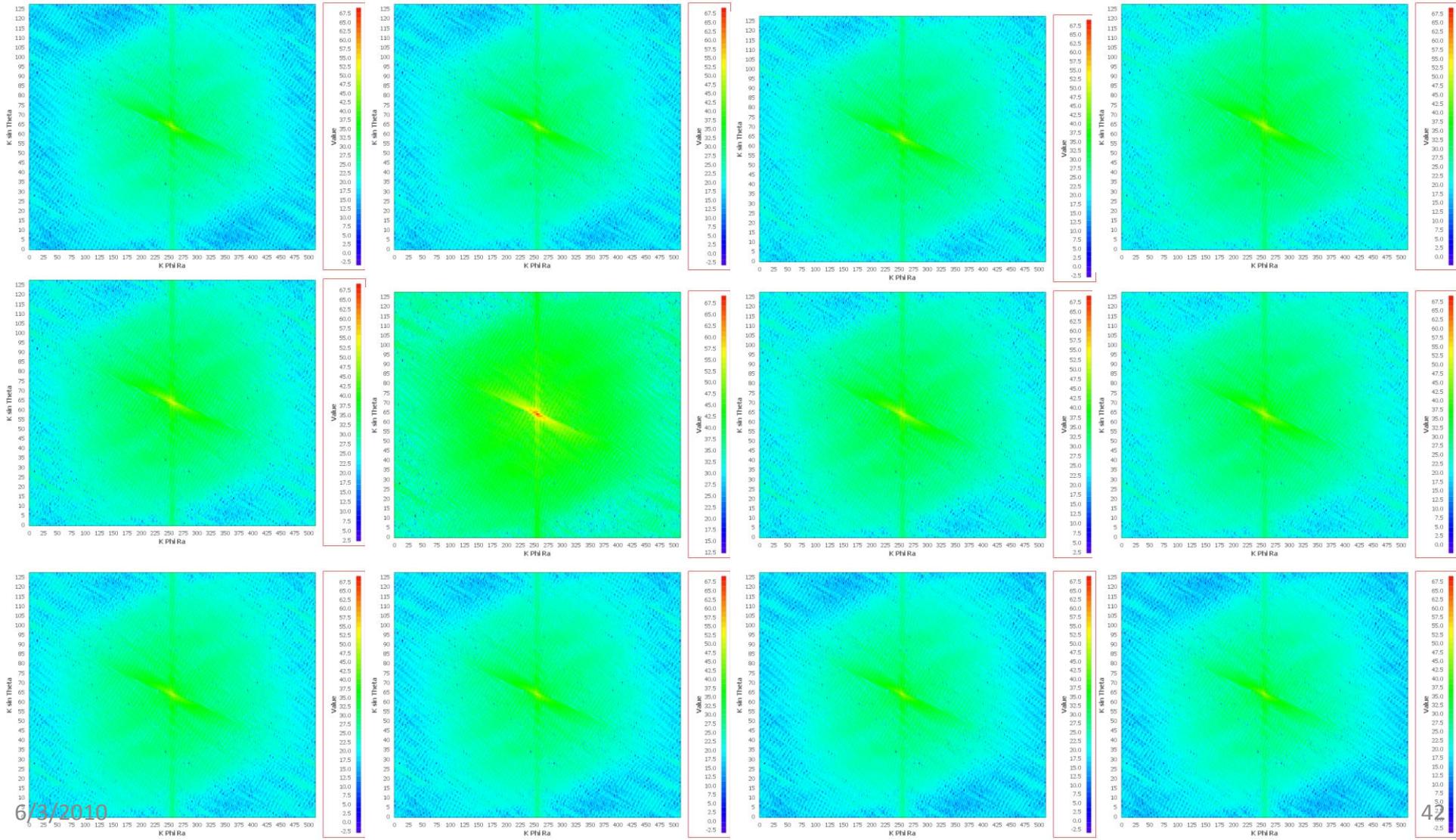


BAO First Peak from 500-1300MHz;  $k_{perp}$  vs “ $k_{||}$ ”; ResBW = 1/128 MHz



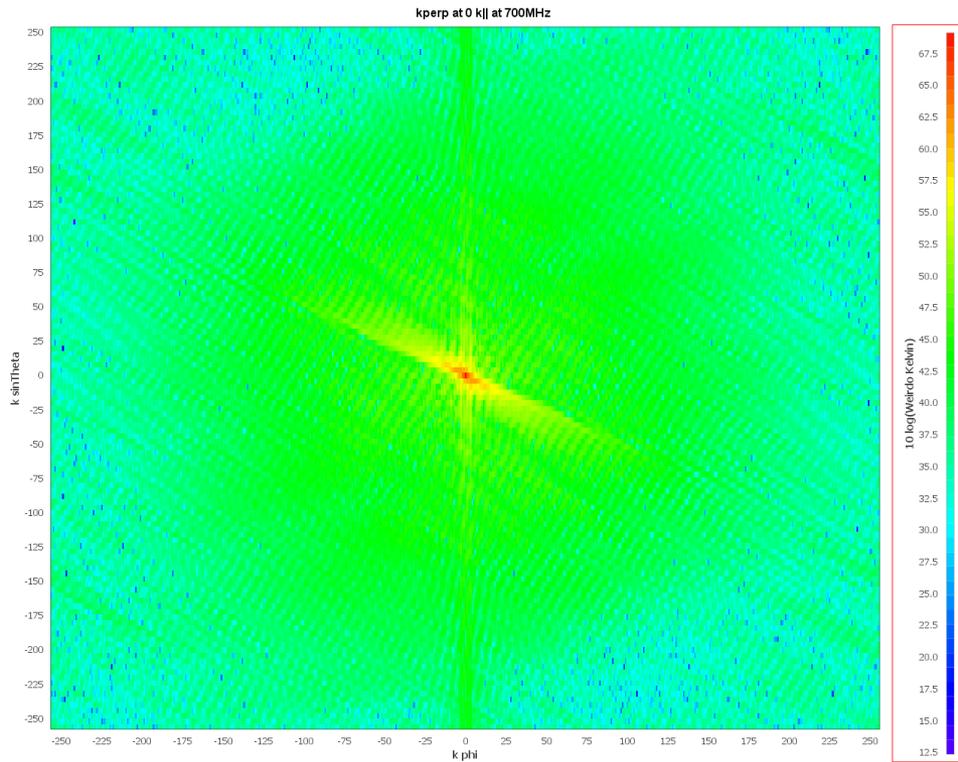
# BAO + Angelica Sky

## ResBW = 1/128 MHz

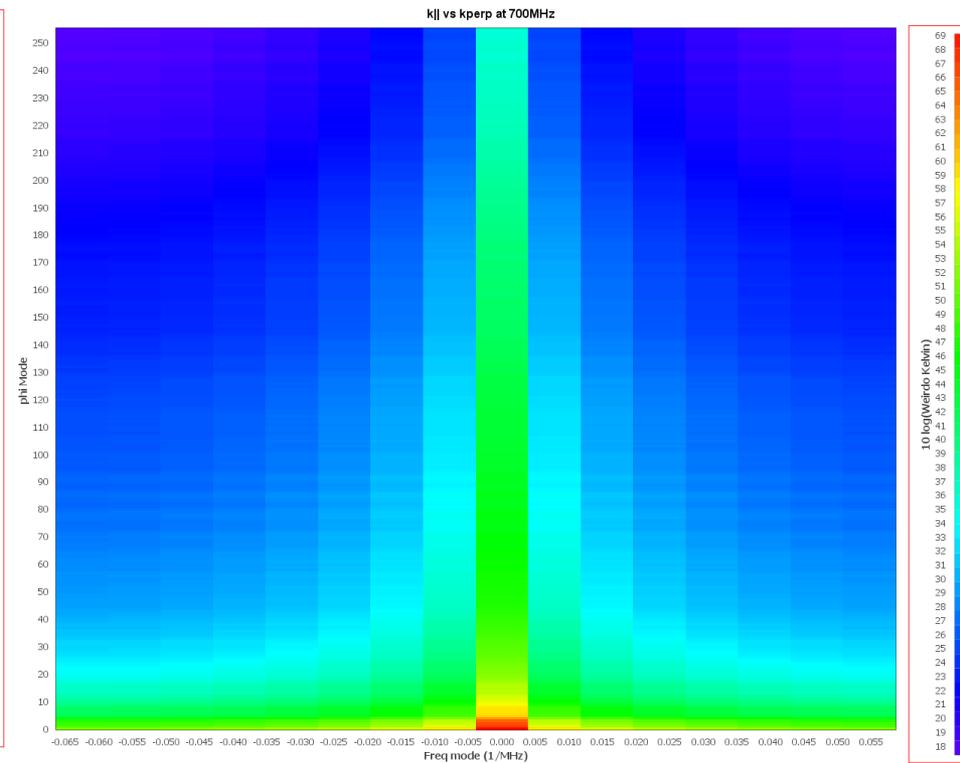


# BAO + Angelica at 700 MHz

## ResBW = 1/128 MHz



kperp at  $k_{\parallel} = 0$  slice



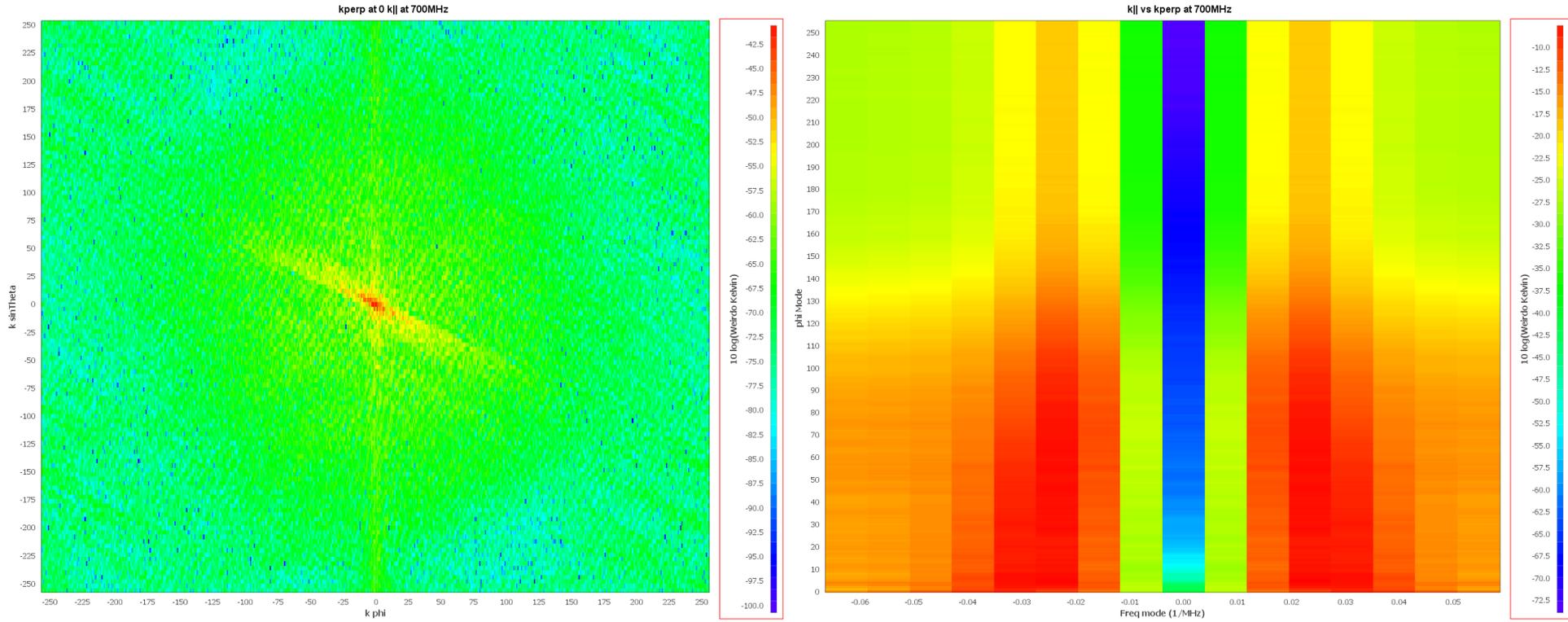
kperp vs “ $k_{\parallel}$ ”

# Frequency-Smoothed Sky Algorithm in Reconstructed K-space

- Work in reconstructed sky transverse k-space
  - Addresses using up polynomial fitting “horsepower” on mode mixing
- Smooth in frequency by fitting an N order polynomial along frequency axis for each transverse k-space pixel
- Subtract frequency smoothed k-space from raw k-space
- Fourier transform along frequency axis
- Look the transverse k-space slices at high  $k_{||}$  mode number.

# BAO + Angelica Sky at 700 MHz with Foreground Removal

## ResBW = 1/128 MHz

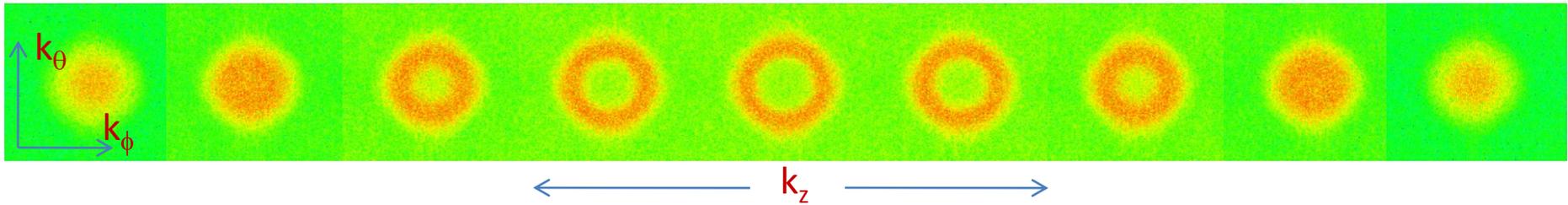


kperp at  $k_{||} = 0$  slice

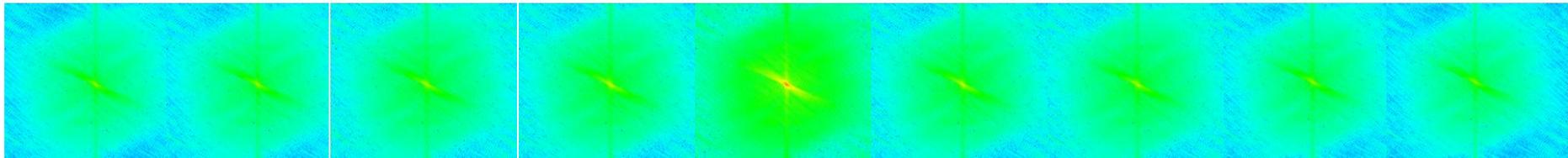
kperp vs " $k_{||}$ "

# Foreground Removal (Fermilab)

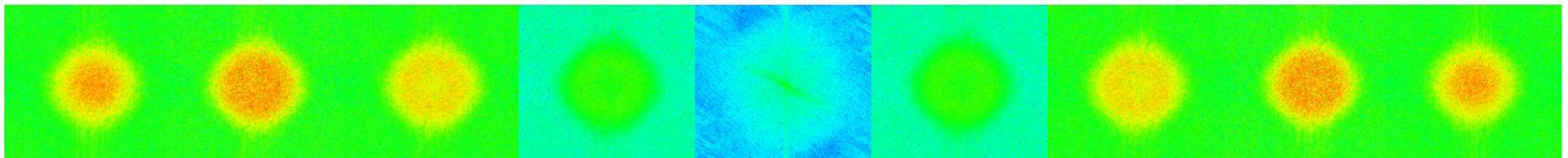
BAO First Peak in 3-D k-Space (Gnedin)



BAO First Peak and Foreground in 3-D k-Space



BAO First Peak and Foreground with Foreground Removal in 3-D k-Space



# Summary

- We have developed fairly sophisticated
  - Instrument modeling software
  - Sky Reconstruction software
  - BAO and foreground sky maps
- We have begun initial tests of foreground removal algorithms
  - Sky model subtraction algorithm on the raw data cube
  - Frequency-smoothed sky subtraction algorithm on the raw data cube
  - Frequency-smoothed sky subtraction algorithm in reconstructed k-space
- Initial results look promising
  - Can remove 5 orders of magnitude of foreground on a raw data cube
  - Can see the first BAO peak behind foregrounds in reconstructed k-space (6 orders of magnitude reduction)

# Future Work

- Get large scale sky reconstruction software working.
- Test foreground removal with k-space reconstruction of a 256 feed/ cylinder telescope
  - Examine the effects of calibration errors
  - Examine the effects of noise
- Add 2<sup>nd</sup> and 3<sup>rd</sup> BAO peaks (i.e complete LSS)
- Test other algorithms

# BACKUP SLIDES

# Grating Lobes

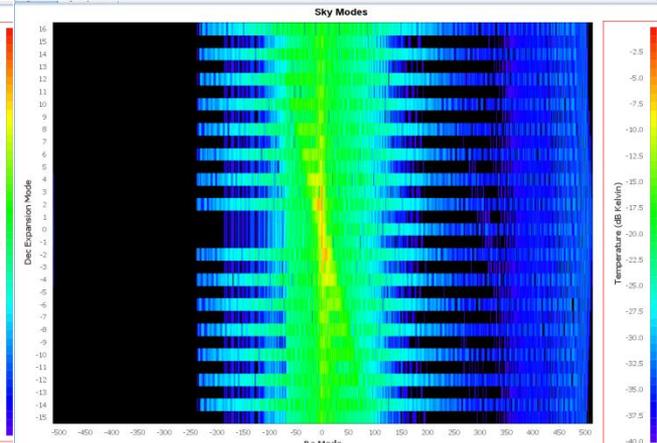
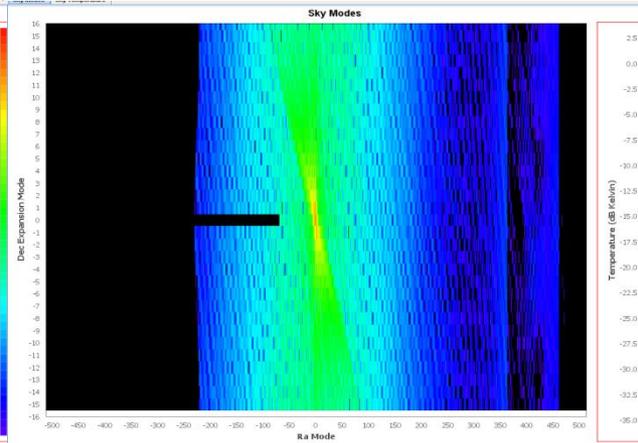
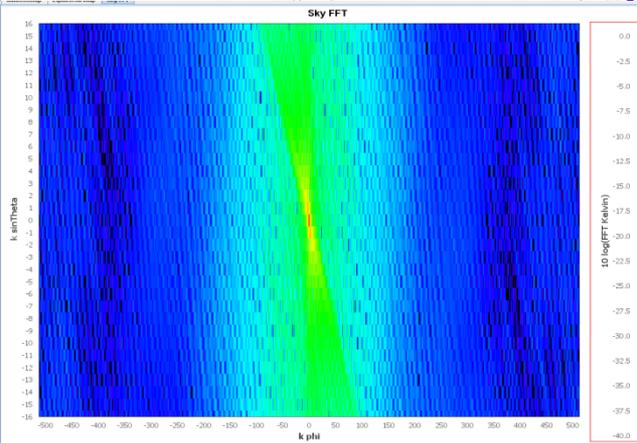
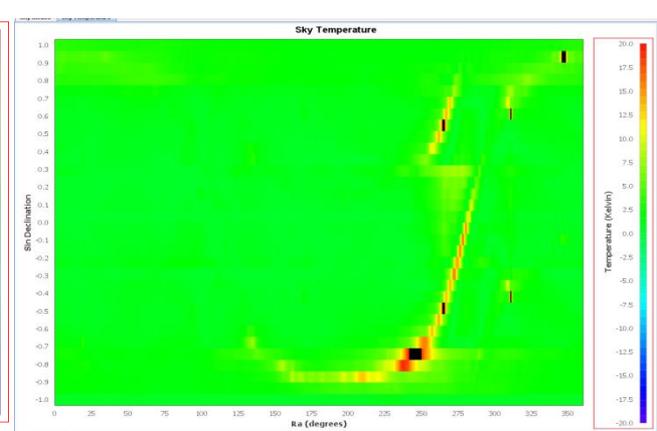
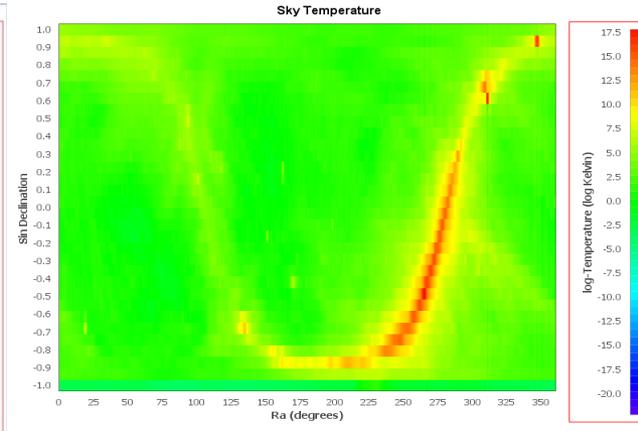
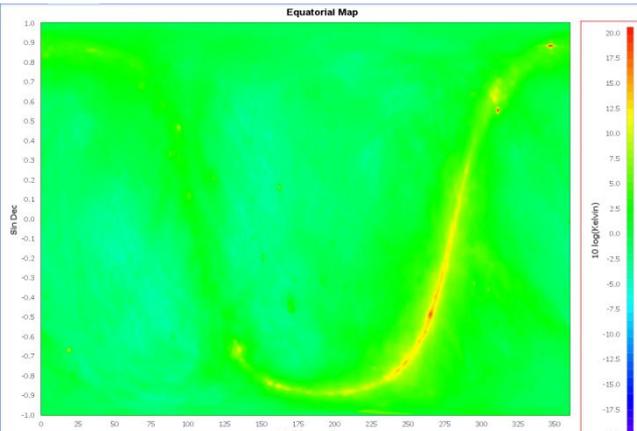
(Pittsburgh Cylinders)

Input Sky

$d/\lambda=0.5$

$d/\lambda=1.0$

Sky Map



K-Space

# Grating Lobes with a Point Source

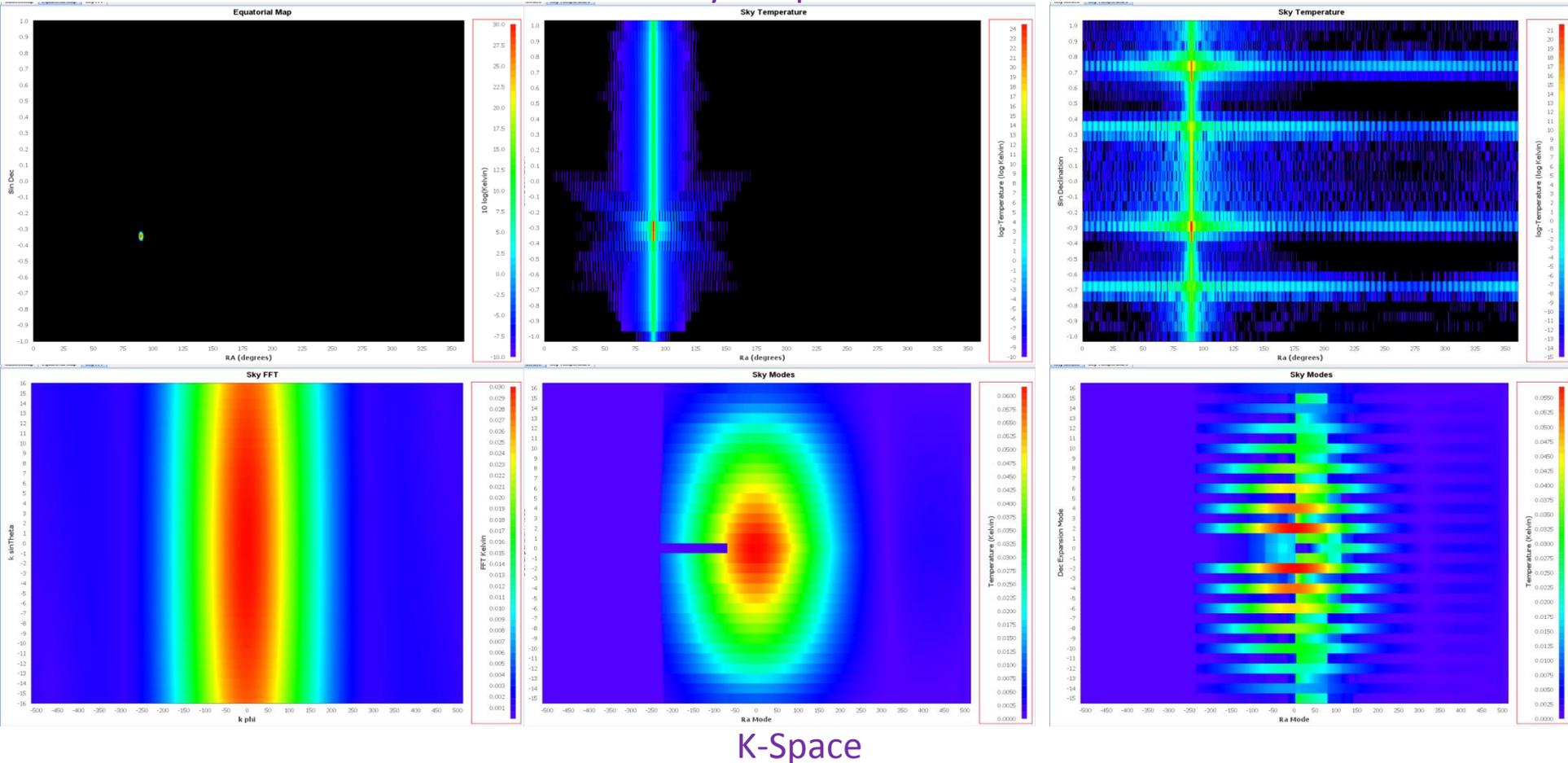
(Pittsburgh Cylinders)

Input Sky

$d/\lambda=0.5$

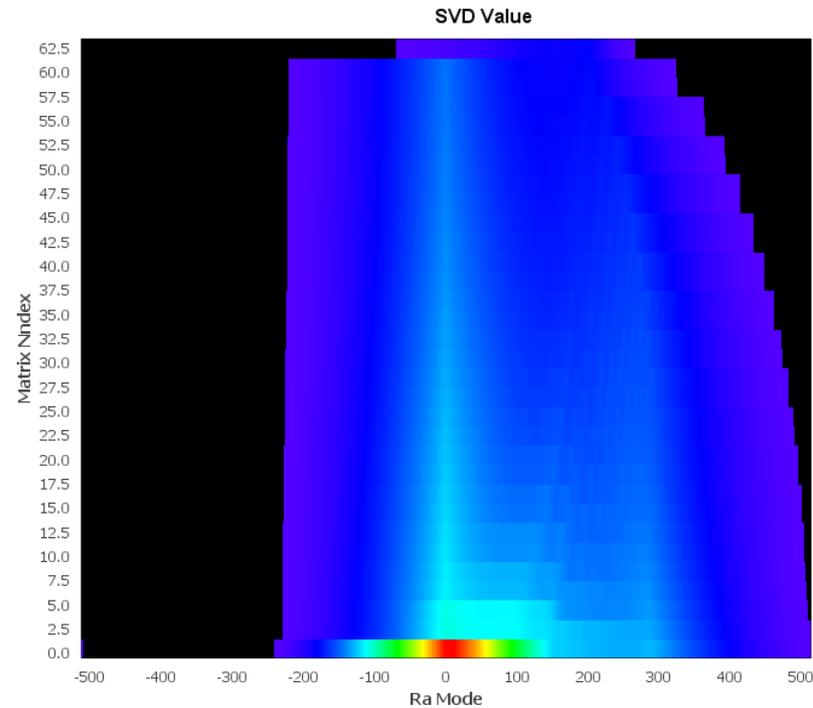
$d/\lambda=1.0$

Sky Map

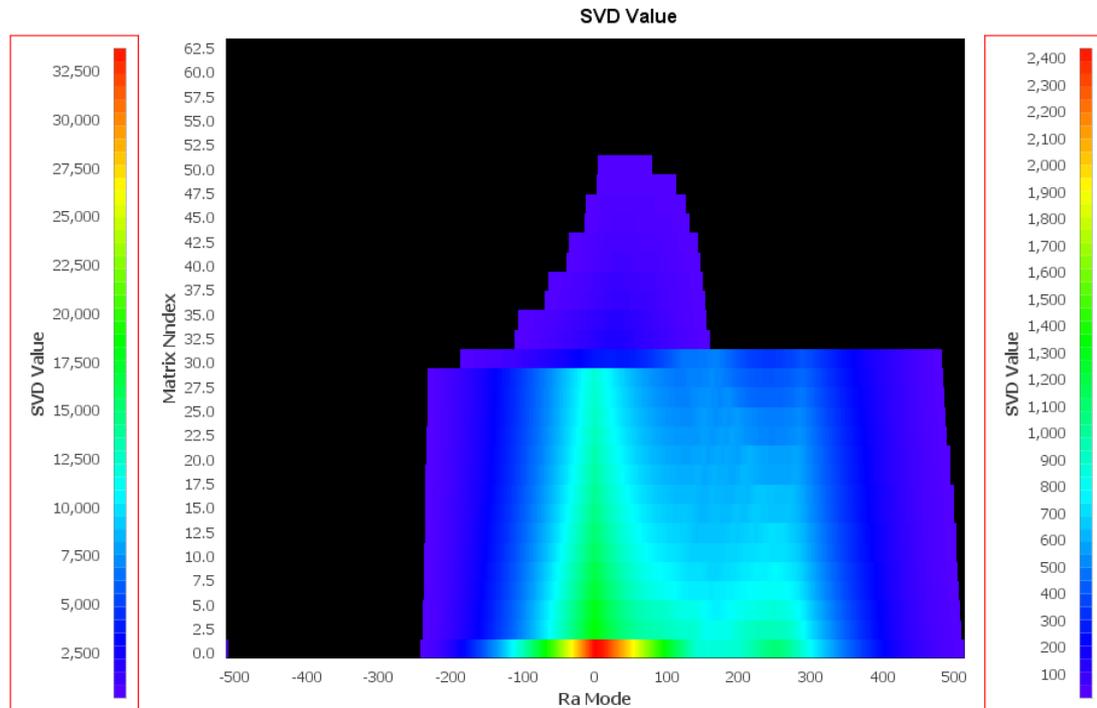


K-Space

# SVD Values of Telescope Model Matrix



$d/\lambda=0.5$



$d/\lambda=1.0$

# Grating Lobes with Sine Wave Sources

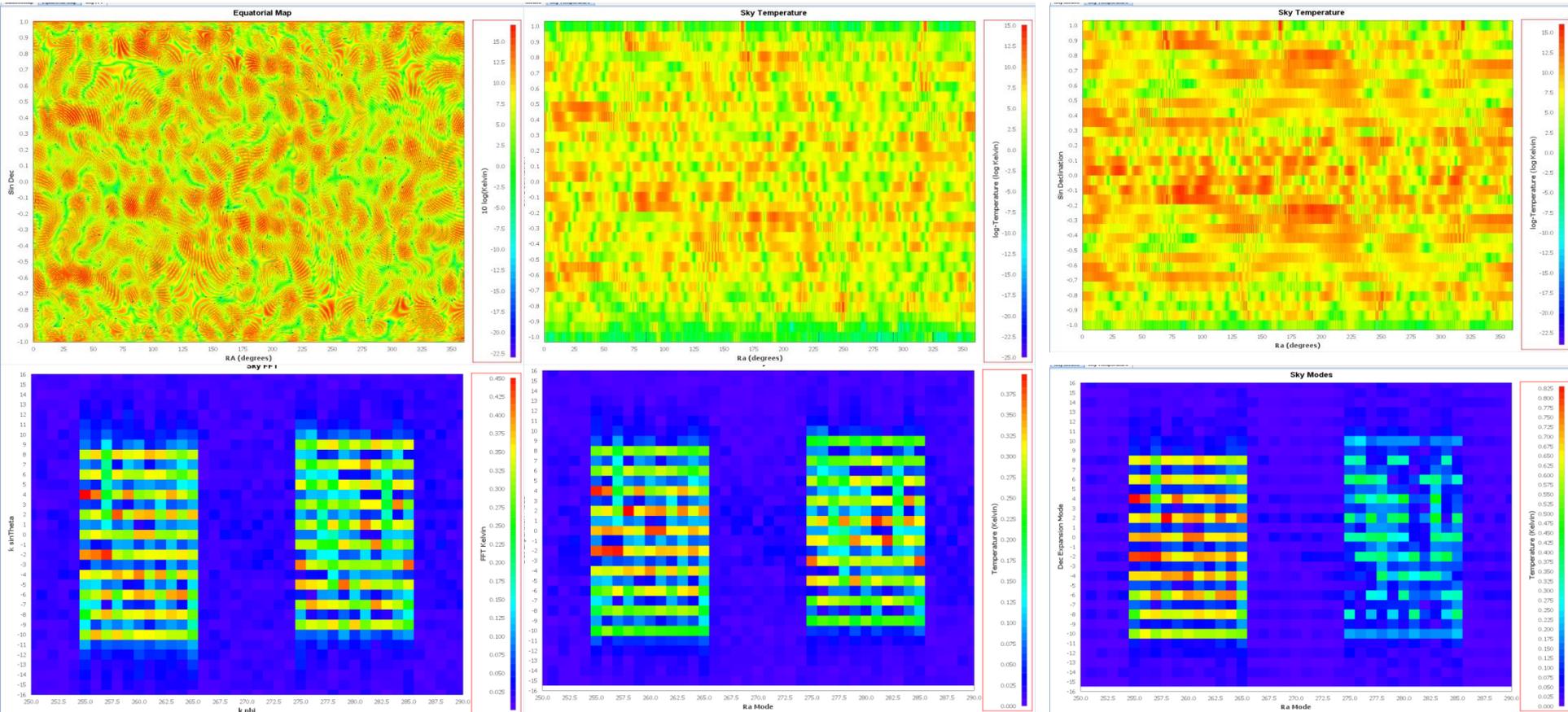
(Pittsburgh Cylinders)

Input Sky

$d/\lambda=0.5$

$d/\lambda=1.0$

Sky Map



K-Space