# Sky Reconstruction from Cylinder Visibilities

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## *Visibility*

This note will consider the reconstruction of the sky from the measured visibilities from a pair of cylinder antenna arrays. It is assumed that the cylinders fixed and are oriented along the meridian. Each cylinder is populated with **N** feeds spaced uniformly along the length. The output voltage of each feed provides an input of a spatial Fourier transform along the cylinder length. The spatial Fourier transform forms **N** beams along the length of the cylinder.

For a pair of cylinders the visibility between cylinders is formed for each beam. As the sky drifts through the cylinder beam, the visibility for beam **k** is:

$$v_k(\varphi) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta,\phi)}{\lambda^2} T(\theta,\varphi-\phi) cos(\theta)\, d\theta d\phi \qquad (1)$$

Where $\boldsymbol{\varphi}$ is the time of the day (in units of angle), $\boldsymbol{\lambda}$ is the wavelength and **T** is the power flux of the sky. The cylinder pair Fourier area is defined as

$$\tilde{A}_k(\theta,\phi) = \tilde{\mathrm{a}}_{\mathrm{k,c1}}(\theta,\phi)\left(\tilde{\mathrm{a}}_{\mathrm{k,c2}}(\theta,\phi)\right)^* \qquad (2)$$

where the subscripts **c1**, **c2** indicate cylinder 1 and cylinder 2, respectively. The Fourier root area of a cylinder is defined as

$$\tilde{\mathrm{a}}_{\mathrm{k,c}}(\theta,\phi) = \sum_n \mathrm{a_n}(\theta,\phi) e^{-j\vec{\beta}(\theta,\phi)\cdot\vec{r}_{n,c}} e^{j2\pi k\frac{n}{N}} \qquad (3)$$

Where **n** is the feed number, $\mathbf{r}_{\mathbf{n,c}}$ is the global location of the feed and $\boldsymbol{\beta}$ is the incoming wave vector:

$$\vec{\beta}(\theta,\phi) = \frac{2\pi}{\lambda}\left(sin(\theta)\hat{x} + cos(\theta)sin(\phi)\hat{y}\right) \qquad (4)$$

It is assumed that the length of the cylinders is in the x direction.

## *Sky Expansion*

Since the sky is periodic, it can be expanded in a Fourier series:

$$T_c(\theta,\phi) = \sum_l \sum_m \hat{T}_{m,l} e^{jl\pi sin(\theta)} e^{jm\phi} \qquad (5)$$

Since $T_c$ is a complex function but the sky temperature must be a real function, the sky temperature can be written as:

$$T(\theta,\phi) = T_c(\theta,\phi) + \left(T_c(\theta,\phi)\right)^* \qquad (6)$$

$$T(\theta,\phi) = \sum_l \sum_m \hat{T}_{m,l} e^{jl\pi sin(\theta)} e^{jm\phi} + \sum_l \sum_m \left(\hat{T}_{m,l}\right)^* e^{-jl\pi sin(\theta)} e^{-jm\phi} \quad (7)$$

$$T(\theta,\phi) = \sum_l \sum_m \left(\hat{T}_{m,l} + \left(\hat{T}_{-m,-l}\right)^*\right) e^{jl\pi sin(\theta)} e^{jm\phi} \quad (8)$$

Substituting Equation 8 into Equation 1,

$$v_k(\varphi) = \sum_l \sum_m \left(\hat{T}_{m,l} + \left(\hat{T}_{-m,-l}\right)^*\right) e^{jm\varphi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta,\phi)}{\lambda^2} e^{jl\pi sin(\theta)} e^{-jm\phi} cos(\theta)\, d\theta d\phi \quad (9)$$

## Fourier Transform of Cylinder Visibilities

Now assume that the visibility is measured at N discrete times during the day. The visibility will be periodic with a period of a day so we can expand the visibility into a Fourier series.

$$v_k(\varphi_n) = \sum_{m'} \tilde{V}_{k,m'} e^{jm\varphi_n} \quad (10)$$

where

$$\tilde{V}_{k,m'} = \frac{1}{N} \sum_n v_k(\varphi_n)\ e^{-jm\varphi_n} \quad (11)$$

Substituting Equations 9 into Equations 11,

$$\tilde{V}_{k,m} = \sum_l \hat{A}_{k,l,m} \left(\hat{T}_{m,l} + \left(\hat{T}_{-m,-l}\right)^*\right) \quad (12)$$

where:

$$\hat{A}_{k,l,m} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta,\phi)}{\lambda^2} e^{jl\pi sin(\theta)} e^{-jm\phi} cos(\theta)\, d\theta d\phi \quad (13)$$

The discrete approximation for Equation 13 is:

$$\hat{A}_{k,l,m} \approx \frac{4\pi}{N_p N_q} \sum_{q=0}^{N_q} \sum_{p=0}^{N_p} \frac{\tilde{A}_k(\theta_p,\phi_q)}{\lambda^2} e^{jl(\pi sin(\theta))_p} e^{-jm\phi_q} \quad (14)$$

The matrix form of Equation 13 is:

$$[\tilde{V}_m] = [\hat{A}_m][\tilde{T}_m] \quad (15)$$

## Multiple Visibilities

There can be a number of combination of cylinder visibilities that have a non-zero component a given right ascension mode m. However, the sky mode temperature at mode m must be unique.

$$[\tilde{V}_m]^{(1)} = [\hat{A}_m]^{(1)}[\tilde{T}_m] \quad (16)$$

and:

$$[\tilde{V}_m]^{(2)} = \left[\hat{A}_m\right]^{(2)}[\tilde{T}_m] \tag{17}$$

To satisfy both equations in a least squares fit, equations 16 and 17 can be combined:

$$\begin{aligned} &\left\{\left\{\left[\hat{A}_m\right]^{(1)}\right\}^T\right\}^* [\tilde{V}_m]^{(1)} + \left\{\left\{\left[\hat{A}_m\right]^{(2)}\right\}^T\right\}^* [\tilde{V}_m]^{(2)} \\ &\qquad = \left(\left\{\left\{\left[\hat{A}_m\right]^{(1)}\right\}^T\right\}^* \left[\hat{A}_m\right]^{(1)} + \left\{\left\{\left[\hat{A}_m\right]^{(2)}\right\}^T\right\}^* \left[\hat{A}_m\right]^{(2)}\right)[\tilde{T}_m] \end{aligned} \tag{18}$$