Comments on the Performance of a Adjacent Feed Summed Phased Array

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Introduction

For the 21cm Cylindrical Radio Telescope, it has been proposed that, to save cost, pairs of feed antenna along the length of the cylinder are analog summed before digitization.[1] The purpose was to reduce the number of digitizing channels which would reduce the size of the sky coverage but not resolution. This note will show that the analog summing of adjacent channels introduces the appearance of strong alias lobes in the antenna beam pattern. Hence, the design strategy of combining adjacent pairs of feeds will probably not be viable.

Analysis

For an array in which adjacent pairs of feed are summed before digitization it is fairly straightforward to show using phased array analysis[2] that the total array voltage is given as

$$\mathrm{V}(\theta,\mathrm{n}) = 2\mathrm{N}\, cos\left(\frac{\pi}{2}\mathrm{x}\right) e^{-j\pi(N-1)\left(x+\frac{n}{N}\right)} \sum_{m=-\infty}^{\infty} (-1)^m\, Sa\left(\pi N\left(x+\frac{n}{N}-m\right)\right) \qquad (1)$$

where:

$$\mathrm{x} = \frac{2d}{\lambda} sin(\theta) \qquad (2)$$

and λ is the wavelength, d is the spacing between feeds, N is the number of digitizing channels, and n is the beam number which ranges from $-N/2 < n < N/2$.

The main lobe is centered at:

$$\sin(\theta_{\mathrm{ml}}) = \frac{n}{N}\frac{\lambda}{2d} \qquad (3)$$

The amplitude of the main lobe is:

$$A_{\mathrm{ml}} = cos\left(\pi\frac{d}{\lambda}\sin(\theta_{\mathrm{ml}})\right) \qquad (4)$$

The first alias side-lobe occurs at:

$$\sin(\theta_{\mathrm{sl}}) = \sin(\theta_{\mathrm{ml}}) - \frac{\lambda}{2d} \qquad (5)$$

The amplitude of the alias side-lobe is given as:

$$A_{\mathrm{sl}} = cos\left(\pi\frac{d}{\lambda}\left(\sin(\theta_{\mathrm{ml}}) - \frac{\lambda}{2d}\right)\right) = sin\left(\pi\frac{d}{\lambda}\sin(\theta_{\mathrm{ml}})\right) \qquad (6)$$

The furthest extent of the main lobe occurs when $n = N/2$

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[1] "The Cylindrical Radio Telescope," Peterson, et.al.," February 2009, http://projects-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=450

[2] "Phased Array Antenna," McGinnis, November 2007, http://projects-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=471

$$\sin(\theta_{ml})_{N/2} = \frac{\lambda}{4d} \tag{7}$$

The amplitude of the main lobe becomes:

$$A_{{ml}_{N/2}} = cos\left(\frac{\pi}{4}\right) \tag{8}$$

The side-lobe is located at:

$$\sin(\theta_{sl})_{N/2} = -\frac{\lambda}{4d} \tag{9}$$

The amplitude of the side-lobe is:

$$A_{{sl}_{N/2}} = sin\left(\frac{\pi}{4}\right) \tag{10}$$

which is equal to the main lobe amplitude. Thus when the main lobe is at its furthest angle from the zenith, the side-lobe is of equal amplitude and is of opposite angle from the zenith.