General Requirement Formulae for the 21cm Cylindrical Radio Telescope

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Frequency

Frequency as a function of redshift:

$$F = \frac{1.4GHz}{1+z} \tag{1}$$

where z is the redshift.

Survey Area and Feed Spacing with Grating Lobes

The survey area for a drift telescope:

$$\mathrm{A} = \int_0^{2\pi} d\phi \int_{-\Delta\theta/2}^{\Delta\theta/2} \cos(\theta)\, d\theta = 4\pi sin\left(\frac{\Delta\theta}{2}\right) \tag{2}$$

where $\Delta\theta$ is the angular span of the telescope beam along the meridian. For an FFT telescope which is oriented along the meridian, the main lobe beam angle is given by:

$$sin(\theta_n) = \frac{n}{N_f}\frac{\lambda}{d} \tag{3}$$

where λ is the wavelength, d is the spacing between digitizers and N is the number of feeds. The angle of the first grating lobe is given as:

$$sin(\theta_{sl}) = sin(\theta_n) - \frac{\lambda}{d} \tag{4}$$

The first grating lobe appears on the horizon when:

$$\frac{\lambda}{\mathrm{d}} = 1 - sin(\theta_{n_{max}}) = 1 + sin\left(\frac{\Delta\theta}{2}\right) \tag{5}$$

For a given survey area, the feed spacing must be:

$$A = 4\pi\left(\frac{1}{d/_{\lambda}} - 1\right) \tag{6}$$

The maximum number of beams that are not aliased is $2n_{max}$ where

$$2n_{max} = 2N_f\left(1 - \frac{d}{\lambda}\right) \tag{7}$$

Survey Area and Feed Spacing without Grating Lobes

If the antennea are summed in such away as to minimize the impact of the grating lobes folding over into the main field of view, then:

$$n_{max} = \frac{N_f}{2} \tag{8}$$

so that:

$$sin(\theta_{n_{max}}) = \frac{1}{2}\frac{\lambda}{d} \tag{9}$$

and:

$$A = 2\pi\frac{\lambda}{d} \tag{10}$$

Angular Resolution Along the Meridian

The length of the cylinder determines the resolution of the beam width along the meridian.

$$\delta\theta = \frac{\lambda}{L_{cyl}} = \frac{\lambda}{N_f d} \tag{11}$$

where $\delta\theta$ is the angular resolution along the meridian, L_{cyl} is the length of the cylinder, N_{feed} is the number of feeds:

$$N_f = \frac{\lambda}{d}\frac{1}{\delta\theta} \tag{12}$$

Redshift and Frequency Resolution

The resolution in redshift determines the depth of the 3-D pixel. An empirical formula for a uniform 3-D pixel is:

$$\delta z \approx 0.436 \times \delta\theta(\text{radians}) \times z(z+2) \tag{13}$$

The frequency resolution is as a function of redshift resolution is:

$$\delta F = \frac{1.4GHz}{(1+z)^2}\delta z \tag{14}$$

which becomes

$$\delta F \approx 610MHz \times \delta\theta(\text{radians}) \times \frac{z(z+2)}{(1+z)^2} \tag{15}$$

Survey Speed and Cylinder Width

The azimuthal angular resolution of a single cylinder is

$$\delta\phi_{cyl} = \frac{\lambda}{W_{cyl}} \tag{16}$$

where W_{cyl} is the width of the cylinder. In one day, the amount of time that an object will sit in the cylinder beam as it drifts across the cylinder beam width is given as:

$$\Delta\tau_{day} = 24 \times 3600\text{sec} \times \frac{{}^{\lambda}/_{W_{cyl}}}{2\pi} \tag{17}$$

The amount of time that can be integrated in one year for an object is:

$$\Delta\tau_{year} = 365 \times D_f \Delta\tau_{day} \tag{18}$$

where D_f is the duty factor of observations. If observing is done only at night, then D_f~50%. The total amount of time integrated is:

$$\Delta\tau_{total} = N_{year}\Delta\tau_{year} \tag{19}$$

where N_{year} is the number of years the survey runs.

A single measurement length is inversely proportional to the resolution bandwidth. The total amount of measurements made is given be the total integration time divided by the measurement length. The total number of measurements, M, is:

$$M = \Delta\tau_{total}\delta F \tag{20}$$

The pixel sensitivity is given as:

$$\delta \mathrm{T} = \frac{1}{\sqrt{\mathrm{M}}}\left(\mathrm{T_{sky}} + \frac{1}{\mathrm{g_a}}\frac{d}{\mathrm{h_f}}\sqrt{\frac{N_f}{(N_f-1)}}\sqrt{\frac{N_{cyl}}{(N_{cyl}-1)}}\left(\frac{\Delta N_{max_{cyl}}+1}{N_{cyl}}\right)\mathrm{T_a}\right) \qquad (21)$$

where g_a is the power efficiency of the antenna, d is the feed spacing, T_a is the equivalent amplifier temperature, T_{sky} is the average sky temperature over the pixel, h_f, is the effective height of the feed, N_{cyl} is the number of actual cylinders, and $\Delta N_{max_{cyl}}$ is the greatest multiple of the unit spacing such that all multiples of the unit spacing $\Delta N_{max_{cyl}}$ are present between pairs of cylinders in the array. The effective height of the feed is defined by the collecting area per feed so that:

$$\mathrm{A_{feed}} = \mathrm{h_f W_{cyl}} \qquad (22)$$

For an infinitely short dipole, $h_f \sim \lambda$.

Number of Cylinders

It will be assumed that it is possible to have $\left(\Delta N_{max_{cyl}}+1\right)$ aligned side by side. Each cylinder forms a beam with azimuthal angular resolution given by Equation 16. Combining the signals from all the cylinders with appropriate phase shift between each cylinder signal will form $\left(\Delta N_{max_{cyl}}+1\right)$ azimuthal beams. The width of any one of the synthesized beams:

$$\delta\phi = \frac{\lambda}{\left(\Delta N_{max_{cyl}}+1\right)\mathrm{W_{cyl}}} \qquad (23)$$

Most likely, the angular resolution in the azimuthal direction, will match the angular resolution in the meridian direction

$$\left(\Delta N_{max_{cyl}}+1\right)\mathrm{W_{cyl}} = N_f d \qquad (24)$$

Example Table Band 1

Number	Requirement	Limit	Average	Limit	Unit
1.01	Redshift Range	1.60	1.17	0.86	
1.02	Survey Area	3.60	3.00	2.57	π steradians
1.03	Angular Resolution	12.1	10.1	8.6	arc-Min
1.04	Survey Time		2.00		Years
1.05	Sensitivity per Pixel	125	183	259	uK
1.06	Polarization Imbalance		-20		dB
2.01	Minumum Antenna efficiency		80		%
2.02	Antenna Width Fill factor		80		%
2.03	Effective Feed Length		1		Wavelength
2.04	Maximum Sky Temperature		10		K
2.05	Maximum Amplifier Noise Temp		50		K
2.06	Observing duty Factor		50		%
2.07	Latitude		0		degrees
2.08	Reflector Cost Rate		7000		$/meter
2.09	Electronics cost per Channel		3000		$/Channel
3.01	Center Frequency		645.0		MHz
3.02	Frequency Span		215.0		MHz
3.03	Frequency	537.5	645.0	752.5	MHz
3.04	Wavelength	55.8	46.5	39.9	cm
3.05	Resolution Bandwidth	1.8	1.4	1.1	MHz
3.06	Integration Time per pixel	2.55	2.13	1.82	day
3.07	Feed Spacing		31.0		cm
3.08	Declination Span	128.4	97.2	80.0	degrees
3.09	Number of Digital Channels per Cylinder per Polarization		512		
3.10	Minimum Digital Memory		395		Samples
3.11	Number of Feed Antenna per Cylinder per Polarization		512		
3.12	Length of Cylinder		158.7		meters
3.13	Width of Cylinder		12.7		meters
3.14	Cylinder Nominal Spacing		15.9		meters
3.15	Number of Cylinders Locations		10.0		
3.16	Number of Actual Cylinders		5.0		
3.17	Number of Channels per Polarization		2560		
3.18	Reflector Cost		5.6		M$
3.19	Electronics Cost		15.4		M$
3.20	Total Cost		20.9		M$

Example Table Band 2

Number	Requirement	Limit	Average	Limit	Unit
1.01	Redshift Range	0.87	0.56	0.33	
1.02	Survey Area	3.60	3.00	2.57	π steradians
1.03	Angular Resolution	12.1	10.1	8.6	arc-Min
1.04	Survey Time		2.00		Years
1.05	Sensitivity per Pixel	137	212	329	uK
1.06	Polarization Imbalance		-20		dB
2.01	Minumum Antenna efficiency		80		%
2.02	Antenna Width Fill factor		80		%
2.03	Effective Feed Length		1		Wavelength
2.04	Maximum Sky Temperature		10		K
2.05	Maximum Amplifier Noise Temp		50		K
2.06	Observing duty Factor		50		%
2.07	Latitude		0		degrees
2.08	Reflector Cost Rate		7000		$/meter
2.09	Electronics cost per Channel		3000		$/Channel
3.01	Center Frequency		900.0		MHz
3.02	Frequency Span		300.0		MHz
3.03	Frequency	750	900.0	1050	MHz
3.04	Wavelength	40.0	33.3	28.6	cm
3.05	Resolution Bandwidth	1.5	1.0	0.7	MHz
3.06	Integration Time per pixel	2.56	2.13	1.83	day
3.07	Feed Spacing		22.2		cm
3.08	Declination Span	128.6	97.3	80.1	degrees
3.09	Number of Digital Channels per Cylinder per Polarization		512		
3.10	Minimum Digital Memory		894		Samples
3.11	Number of Feed Antenna per Cylinder per Polarization		512		
3.12	Length of Cylinder		113.7		meters
3.13	Width of Cylinder		9.1		meters
3.14	Cylinder Nominal Spacing		11.4		meters
3.15	Number of Cylinders Locations		10.0		
3.16	Number of Actual Cylinders		5.0		
3.17	Number of Channels per Polarization		2560		
3.18	Reflector Cost		4.0		M$
3.19	Electronics Cost		15.4		M$
3.20	Total Cost		19.3		M$