# General Requirement Formulae for the 21cm Cylindrical Radio Telescope

Dave McGinnis
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## Frequency

Frequency as a function of redshift:

$$F = \frac{1.4GHz}{1+z} \tag{1}$$

where z is the redshift.

## Survey Area and Feed Spacing with Grating Lobes

The survey area for a drift telescope:

$$\mathrm{A} = \int_0^{2\pi} d\phi \int_{-\Delta\theta/2}^{\Delta\theta/2} \cos(\theta)\, d\theta = 4\pi sin\left(\frac{\Delta\theta}{2}\right) \tag{2}$$

where $\Delta\theta$ is the angular span of the telescope beam along the meridian. For an FFT telescope which is oriented along the meridian, the main lobe beam angle is given by:

$$sin(\theta_n) = \frac{n}{N_{feed}} \frac{\lambda}{d} \tag{3}$$

where $\lambda$ is the wavelength, d is the spacing between digitizers and N is the number of feeds. The angle of the first grating lobe is given as:

$$sin(\theta_{sl}) = sin(\theta_n) - \frac{\lambda}{d} \tag{4}$$

The first grating lobe appears on the horizon when:

$$\frac{\lambda}{\mathrm{d}} = 1 - sin\left(\theta_{n_{max}}\right) = 1 + sin\left(\frac{\Delta\theta}{2}\right) \tag{5}$$

For a given survey area, the feed spacing must be:

$$A = 4\pi\left(\frac{1}{d/_{\lambda}} - 1\right) \tag{6}$$

The maximum number of beams that are not aliased is $2n_{max}$ where

$$2n_{max} = 2N_{feed}\left(1 - \frac{d}{\lambda}\right) \tag{7}$$

## Survey Area and Feed Spacing without Grating Lobes

If the antennea are summed in such away as to minimize the impact of the grating lobes folding over into the main field of view, then:

$$n_{max} = \frac{N_{feed}}{2} \tag{8}$$

so that:

$$sin\left(\theta_{n_{max}}\right) = \frac{1}{2}\frac{\lambda}{d} \tag{9}$$

and:

$$A = 2\pi \frac{\lambda}{d} \tag{10}$$

## *Angular Resolution Along the Meridian*

The length of the cylinder determines the resolution of the beam width along the meridian.

$$\delta\theta = \frac{\lambda}{L_{cyl}} = \frac{\lambda}{N_{feed} d} \tag{11}$$

where $\delta\theta$ is the angular resolution along the meridian, $L_{cyl}$ is the length of the cylinder, $N_{feed}$ is the number of feeds:

$$N_{feed} = \frac{\lambda}{d} \frac{1}{\delta\theta} \tag{12}$$

## *Redshift and Frequency Resolution*

The resolution in redshift determines the depth of the 3-D pixel. An empirical formula for a uniform 3-D pixel is:

$$\delta z \approx 0.436 \times \delta\theta(\text{radians}) \times z(z + 2) \tag{13}$$

The frequency resolution is as a function of redshift resolution is:

$$\delta F = \frac{1.4GHz}{(1 + z)^2} \delta z \tag{14}$$

which becomes

$$\delta F \approx 610MHz \times \delta\theta(\text{radians}) \times \frac{z(z + 2)}{(1 + z)^2} \tag{15}$$

## *Survey Speed and Cylinder Width*

The azimuthal angular resolution of a single cylinder is

$$\delta\phi_{cyl} = \frac{\lambda}{W_{cyl}} \tag{16}$$

where $W_{cyl}$ is the width of the cylinder. In one day, the amount of time that an object will sit in the cylinder beam as it drifts across the cylinder beam width is given as:

$$\Delta\tau_{day} = 24 \times 3600\text{sec} \times \frac{\lambda / W_{cyl}}{2\pi} \tag{17}$$

The amount of time that can be integrated in one year for an object is:

$$\Delta\tau_{year} = 365 \times D_f \Delta\tau_{day} \tag{18}$$

where $D_f$ is the duty factor of observations. If observing is done only at night, then $D_f$~50%. The total amount of time integrated is:

$$\Delta\tau_{total} = N_{year} \Delta\tau_{year} \tag{19}$$

where $N_{year}$ is the number of years the survey runs.

A single measurement length is inversely proportional to the resolution bandwidth. The total amount of measurements made is given be the total integration time divided by the measurement length. The total number of measurements, M, is:

$$M = \Delta\tau_{total} \delta F \tag{20}$$

The pixel sensitivity is given as:

$$\delta T = \frac{1}{\sqrt{M}}\left(\frac{1}{g_a} T_a + T_{sky}\right) \tag{21}$$

where $g_a$ is the power efficiency of the antenna, $T_a$ is the equivalent amplifier temperature, and $T_{sky}$ is the average sky temperature over the pixel. Combining Equations 16-21, the required cylinder width is:

$$W_{cyl} = \lambda\left(\frac{g_a \delta T}{T_a + g_a T_{sky}}\right)^2 D_f N_{year} \delta F \frac{365 \times 24 \times 3600 \text{ Sec}}{2\pi} \tag{22}$$

The cylinder width cannot very as a function of frequency. Since the resolution of the cylinder is a function of frequency, then the total integration time for a single pixel will be a function of frequency.

$$\Delta\tau_{total} = \frac{\lambda}{\lambda_{ref}}\left(\frac{T_a + g_a T_{sky}}{g_a \delta T_{ref}}\right)^2 \frac{1}{\delta F_{ref}} \tag{23}$$

where **ref** denotes values at the reference frequency for which the cylinder width was chosen. Because the total integration time is a function of frequency, the pixel sensitivity will also be a function of frequency:

$$\delta T = \delta T_{ref}\sqrt{\frac{\lambda_{ref}}{\lambda}\frac{\delta F_{ref}}{\delta F}} \tag{24}$$

## *Number of Cylinders*

It will be assumed that there are $N_{cyl}$ aligned side by side. Each cylinder forms a beam with azimuthal angular resolution given by Equation 16. Combining the signals from all the cylinders with appropriate phase shift between each cylinder signal will form $N_{cyl}$ azimuthal beams. The width of any one of the synthesized beams:

$$\delta\phi = \frac{\lambda}{N_{cyl} W_{cyl}} \tag{25}$$

## *Example Table Band 1*

| Number | Requirement | Limit | Average | Limit | Unit | Parent Requirements | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.01 | Redshift Range | 2.11 | 1.59 | 1.22 | | | | | | | | | |
| 1.02 | Survey Area | 2.39 | 1.99 | 1.71 | π steradians | | | | | | | | |
| 1.03 | Angular Resolution | 16.0 | 13.4 | 11.5 | arc-Min | | | | | | | | |
| 1.04 | Survey Time | | 0.87 | | Years | | | | | | | | |
| 1.05 | Sensitivity per Pixel | 142.1 | 175.0 | 210.9 | uK | | | | | | | | |
| 1.06 | Polarization Imbalance | | -20 | | dB | | | | | | | | |
| | | | | | | | | | | | | | |
| 2.01 | Minumum Antenna efficiency | | 80 | | % | | | | | | | | |
| 2.02 | Maximum Sky Temperature | | 10 | | K | | | | | | | | |
| 2.03 | Maximum Amplifier Noise Temp | | 50 | | K | | | | | | | | |
| 2.04 | Observing duty Factor | | 50 | | % | | | | | | | | |
| 2.05 | Latitude | | 0 | | degrees | | | | | | | | |
| | | | | | | | | | | | | | |
| 3.01 | Center Frequency | | 540.0 | | MHz | 1.01 | | | | | | | |
| 3.02 | Frequency Span | | 180.0 | | MHz | 1.01 | | | | | | | |
| 3.03 | Resolution Bandwidth | 2.6 | 2.0 | 1.6 | MHz | 1.01 | 1.03 | | | | | | |
| 3.04 | Integration Time per pixel | 1.18 | 0.98 | 0.84 | | 1.05 | 2.01 | 2.02 | 2.03 | 3.03 | | | |
| 3.05 | Feed Spacing | | 55.8 | | cm | 1.02 | 3.01 | | | | | | |
| 3.06 | Declination Span | 73.4 | 59.7 | 50.5 | degrees | 1.02 | | | | | | | |
| 3.07 | Number of Digital Channels per Cylinder per Polarization | | 256 | | | 1.03 | 3.01 | 3.05 | | | | | |
| 3.08 | Number of Feed Antenna per Cylinder per Polarization | | 512 | | | 3.07 | | | | | | | |
| 3.09 | Length of Cylinder | | 142.8 | | meters | 3.05 | 3.07 | | | | | | |
| 3.10 | Width of Cylinder | | 14.3 | | meters | 1.04 | 1.05 | 2.01 | 2.02 | 2.03 | 2.04 | 3.01 | 3.03 |
| 3.11 | Number of cylinders | | 10.0 | | | 1.03 | 3.01 | 3.10 | | | | | |
| 3.12 | Number of Channels per Polarization | | 2557 | | | 3.07 | 3.11 | | | | | | |

## *Example Table Band 2*

| Number | Requirement | Limit | Average | Limit | Unit | Parent Requirements | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.01 | Redshift Range | 1.24 | 0.87 | 0.60 | | | | | | | | | |
| 1.02 | Survey Area | 2.46 | 2.05 | 1.76 | $\pi$ steradians | | | | | | | | |
| 1.03 | Angular Resolution | 16.5 | 13.8 | 11.8 | arc-Min | | | | | | | | |
| 1.04 | Survey Time | | 1.40 | | Years | | | | | | | | |
| 1.05 | Sensitivity per Pixel | 137.6 | 175.0 | 220.8 | uK | | | | | | | | |
| 1.06 | Polarization Imbalance | | -20 | | dB | | | | | | | | |
| | | | | | | | | | | | | | |
| 2.01 | Minumum Antenna efficiency | | 80 | | % | | | | | | | | |
| 2.02 | Maximum Sky Temperature | | 10 | | K | | | | | | | | |
| 2.03 | Maximum Amplifier Noise Temp | | 50 | | K | | | | | | | | |
| 2.04 | Observing duty Factor | | 50 | | % | | | | | | | | |
| 2.05 | Latitude | | 0 | | degrees | | | | | | | | |
| | | | | | | | | | | | | | |
| 3.01 | Center Frequency | | 750.0 | | MHz | 1.01 | | | | | | | |
| 3.02 | Frequency Span | | 250.0 | | MHz | 1.01 | | | | | | | |
| 3.03 | Resolution Bandwidth | 2.3 | 1.7 | 1.3 | MHz | 1.01 | 1.03 | | | | | | |
| 3.04 | Integration Time per pixel | 1.37 | 1.14 | 0.98 | | 1.05 | 2.01 | 2.02 | 2.03 | 3.03 | | | |
| 3.05 | Feed Spacing | | 39.0 | | cm | 1.02 | 3.01 | | | | | | |
| 3.06 | Declination Span | 76.0 | 61.7 | 52.2 | degrees | 1.02 | | | | | | | |
| 3.07 | Number of Digital Channels per Cylinder per Polarization | | 256 | | | 1.03 | 3.01 | 3.05 | | | | | |
| 3.08 | Number of Feed Antenna per Cylinder per Polarization | | 512 | | | 3.07 | | | | | | | |
| 3.09 | Length of Cylinder | | 99.8 | | meters | 3.05 | 3.07 | | | | | | |
| 3.10 | Width of Cylinder | | 14.3 | | meters | 1.04 | 1.05 | 2.01 | 2.02 | 2.03 | 2.04 | 3.01 | 3.03 |
| 3.11 | Number of cylinders | | 7.0 | | | 1.03 | 3.01 | 3.10 | | | | | |
| 3.12 | Number of Channels per Polarization | | 1787 | | | 3.07 | 3.11 | | | | | | |