General Requirement Formulae for the 21cm Cylindrical Radio Telescope

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Frequency:

$$F(z) = \frac{1.4GHz}{1+z} \tag{1}$$

Frequency resolution:

$$\Delta F(z,\Delta z) = F(z)\frac{\Delta z}{1+z} \tag{2}$$

where Δz is the required resolution in redshift.

Feed Spacing[1]

It is assumed that the feed antenna will have a wide field of view, especially for the polarization perpendicular to the length of the cylinder. For a phased array in which each feed is digitized separately, the first alias will show up on the horizon when the feed spacing d is given as:

$$d(\Delta\theta_\mathrm{D},\mathrm{z}) = \frac{\lambda\big(F(z)\big)}{1+\sin\left(\frac{\Delta\theta_\mathrm{D}}{2}\right)} \tag{3}$$

where λ is the wavelength at the required redshift, $\Delta\theta_D$ is the span in declination that the telescope must cover.

Number of Feeds[1]

$$N_{feed}(\Delta\theta_\mathrm{D},\mathrm{z},\delta\theta_D) = \frac{\lambda\big(F(z)\big)}{d(\Delta\theta_\mathrm{D},\mathrm{z})\ \delta\theta_D} \tag{4}$$

where $\delta\theta_D$ is the pixel size in declination.

Cylinder Length:

$$L(\Delta\theta_\mathrm{D},\mathrm{z},\delta\theta_D) = N_{feed}(\Delta\theta_\mathrm{D},\mathrm{z},\delta\theta_D)\,d(\Delta\theta_\mathrm{D},\mathrm{z}) \tag{5}$$

[1] "Phased Array Antenna," D. McGinnis, November 2007, http://projects-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=471

Cylinder Width[2]

Since the sky will scan through the antenna beam of a cylinder, the width of a cylinder is determined by the amount of integration time needed to obtain the necessary amplitude resolution:

$$W_{cyl}(\tau_s, z, \Delta z, \Delta T_s, T_s, g, T_a) = \frac{1}{2\pi}\tau_s \Delta F(z, \Delta z)\lambda\big(F(z)\big)\left(\frac{\Delta T_s}{T_s + \frac{1}{g}T_a}\right)^2 \qquad (6)$$

where τ_s is the total integration time of the survey, ΔT_s is the amplitude resolution, T_s is the average sky temperature, g is the antenna efficiency, and T_a is the amplifier noise temperature.

Number of Cylinders

The number of cylinders is determined by the resolution required in right ascension

$$N_{cyl}(\tau_s, z, \Delta z, \Delta T_s, T_s, g, T_a) = \frac{\lambda}{\Delta\phi_R W_{cyl}(\tau_s, z, \Delta z, \Delta T_s, T_s, g, T_a)} \qquad (7)$$

where $\Delta\phi_R$ is the required resolution in right ascension.

Amplifier Gain and Phase Variation.

If the gains of the feeds are not calibrated against each other, a point source will not be localized in the pixel map. The level of pixel noise referenced to the signal level of a point source is given as:

$$\frac{\Delta P_{rms}}{P_{psf}} = \frac{\Delta g_{rms}}{\sqrt{N_{feed}}} \qquad (8)$$

where Δg_{rms} is the rms variation in gain in the amplifiers along a cylinder. Similarly for phase noise

$$\frac{\Delta P_{rms}}{P_{psf}} = \frac{\Delta \psi_{rms}}{\sqrt{N_{feed}}} \qquad (9)$$

where $\Delta\psi_{rms}$ is the rms variation in phase in the amplifiers along a cylinder.

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[2] "Integration Time for 21cm Parabolic Cylinder Radio Telescope," D. McGinnis, April 2009, http://projects-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=469