Phased Array Antenna

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Introduction

This note describes the radiation pattern of a one dimensional phased array antenna consisting of N receivers.

The frequency content of the amplitude a(t,θ) hitting the antenna array is:

$$A(\omega,\theta,t) = \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} a(\tau,\theta) e^{-j\omega\tau} d\tau \tag{1}$$

The frequency resolution is given as:

$$\Delta\omega = {}^{1}/_{T} \tag{2}$$

The spacing between receiver elements is d. For a wave impinging on the cylinder at angle θ from directly overhead, the delay of the wavefront hitting element n with respect to the first element in the array is

$$\Delta\tau = n\frac{d}{c}sin(\theta) \tag{3}$$

This delay produces a phase shift at frequency ω:

$$\Delta\psi = -\omega\Delta\tau = -2\pi n\frac{d}{\lambda}sin(\theta) \tag{4}$$

The complex phasor voltage at produced at element n is

$$\mathrm{V_n}(\omega,\theta,t) = \mathrm{g}\, A(\omega,\theta,t) e^{-j2\pi n\frac{d}{\lambda}sin(\theta)} \tag{5}$$

where g is the antenna gain which is assume to be independent of θ.

For a phased array, all the elements are summed together but before the sum is made, an electronic phase shift is added to each receiver. The relative phase shift between receivers is given as ϕ. The total phasor voltage of the N receivers is:

$$\mathrm{V}(\omega,\theta,t) = \mathrm{g}\, A(\omega,\theta,t) \sum_{n=0}^{N-1} e^{-j2\pi n\left(\frac{d}{\lambda}sin(\theta)+\frac{\phi}{2\pi}\right)} \tag{6}$$

which can be re-written as:

$$\mathrm{V}(\omega,\theta,t) = 2\mathrm{g}\, A(\omega,\theta,t) e^{-j\pi(N-1)x} \sum_{k=1}^{N/2} cos\big((2k-1)\pi x\big) \tag{7}$$

where:

$$x = \frac{d}{\lambda} sin(\theta) + \frac{\phi}{2\pi} \tag{8}$$

The sum in Equation 7 is just the Fourier series of the sampling function (Sa(x) = sin(x)/x)

$$\sum_{k=1}^{N/2} cos\big((2k-1)\pi x\big) = \frac{N}{2} \sum_{m=-\infty}^{\infty} (-1)^m \, Sa\big(\pi N(x-m)\big) \tag{9}$$

By changing the electronic phase shift between receivers, the reception pattern can be steered to any angle θ, The nulls in the beam width is given by x=+1/N and x=-1/N. This implies for uniform converge of the sky, the minimum step size in electronic phase shift between receivers is

$$\Delta\phi = \frac{2\pi}{N} \tag{10}$$

Because the phase shift cannot be greater than 2π, the number of beams required to cover the sky is equal to the number of elements. Figure 1 shows the possible power (voltage squared) radiation pattern for 6 receivers. Figure 2 shows the same pattern as a function of sin(θ). Also displayed in Figure 2 in the black trace is the sum of the power radiation patterns. With the step size in phase shift given by Equation 10, the sky coverage is uniform. The phase shifts used to form each beam is -150°,-90°,-30°,30°,90°,150°.

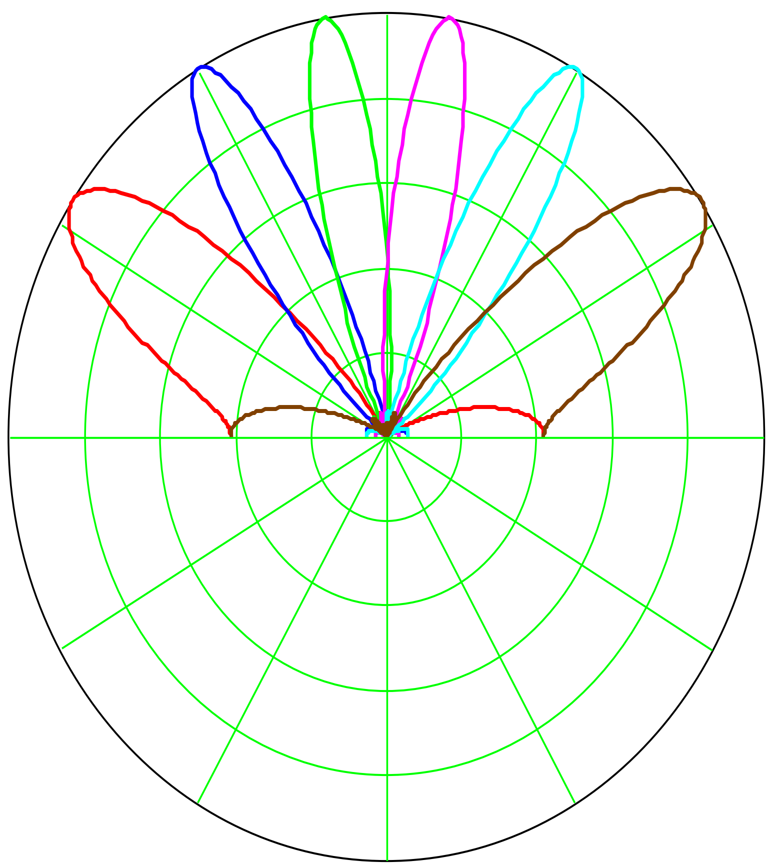

Figure 1. Radiation pattern of phased array with six different electronic phase shifts.
Red = -150°,Blue = -90°, Green = -30°,Magenta = 30°,Cyan = 90°,Brown = 150°.

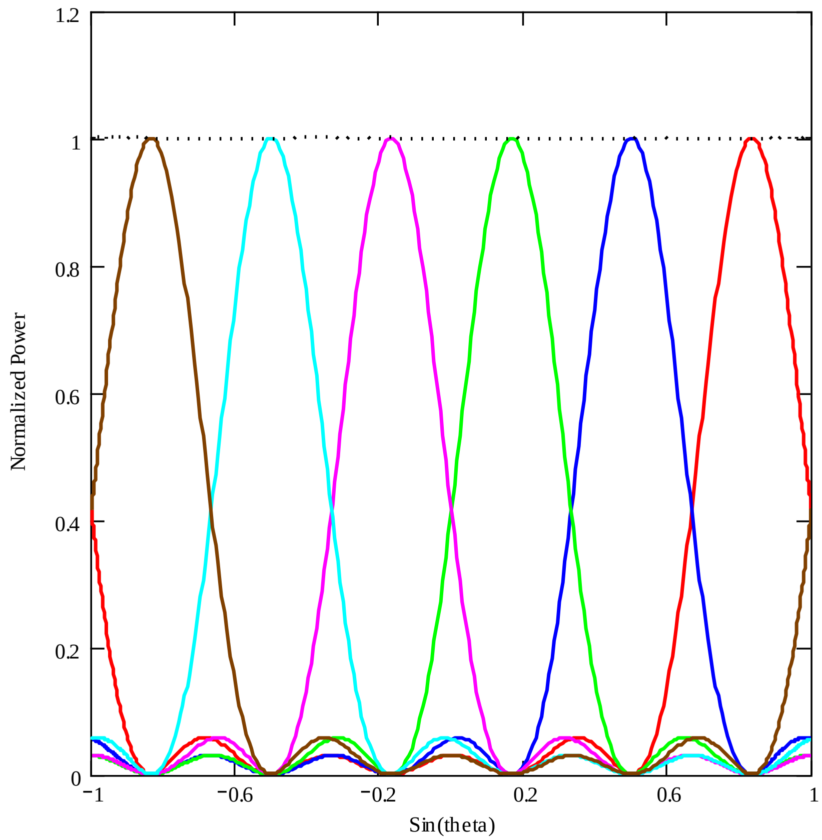

Figure 2. Radiation pattern of phased array with six different electronic phase shifts. Red = -150°,Blue = -90°, Green = -30°,Magenta = 30°,Cyan = 90°,Brown = 150°. The black trace is the sum of all the patterns.

Because sin(θ) can only range between -1 to +1, and if each receiver has no angular dependence (g is independent of θ), Eqn. 9 implies that if d/λ is greater than 0.5 an alias lobe will show up in the radiation pattern. Figure 3 shows the radiation pattern for 6 receivers with an electronic phase shift of -150° between receivers with various ratios of d/λ.

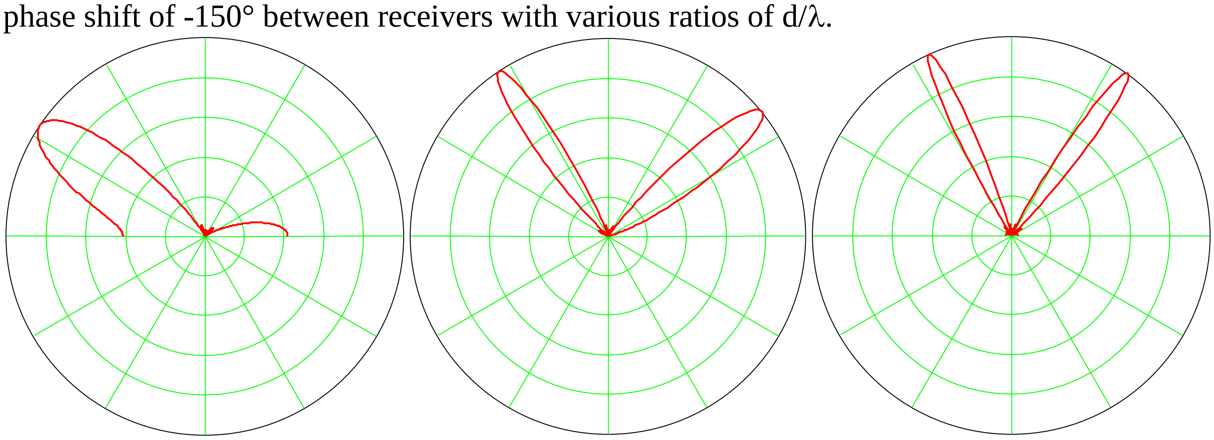

Figure 3. Radiation pattern for 6 receivers with an electronic phase shift of -150° between receivers with various ratios of d/λ. Left plot d/λ=0.5; Center plot d/λ=0.75; Right plot d/λ=1.0

To avoid this alias lobe, d/λ should be chosen to be 0.5 at the highest frequency of the receiver bandwidth. If the bandwidth of the receivers is an octave, then d/λ will be 0.25 at the lowest frequency of the receiver bandwidth. Figure 4 shows the possible radiation patterns for 6 receivers with d/λ =0.25 and d/λ =0.5. As shown in Figure 4, lowering d/λ reduces the resolution of the array.

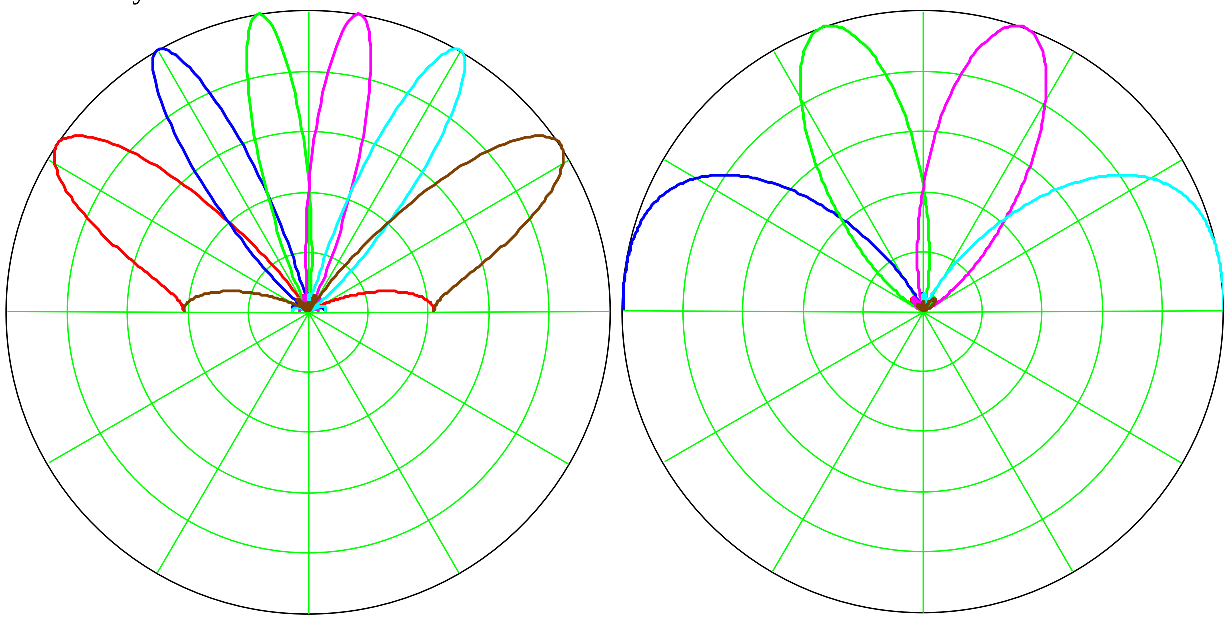

Figure 4. Radiation pattern of phased array with six different electronic phase shifts. Red = -150°,Blue = -90°, Green = -30°,Magenta = 30°,Cyan = 90°,Brown = 150°. Left plot d/λ =0.5. Right plot d/λ =0.25.

The frequency content from each receiver n is:

$$V_n(\omega, t) = \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} v_n(\tau) e^{-j\omega\tau} d\tau \tag{11}$$

The total voltage from the array with a given phase shift between receivers fm is

$$\mathrm{V}(\omega, t, m) = \sum_{n=0}^{N-1} V_n(\omega, t) e^{jn\phi_m} \tag{12}$$

Where using Equation 10:

$$\phi_{\mathrm{m}} = \phi_o + \frac{2\pi}{N} m \tag{13}$$

Using Equation 6, Equation 12 becomes:

$$\sum_{n=0}^{N-1} V_n(\omega, t) e^{jn\phi_m} = \mathrm{g}\, A(\omega, \theta(z), t) \sum_{n=0}^{N-1} e^{-j2\pi n\left(z - \frac{\phi_m}{2\pi}\right)} \tag{14}$$

where:

$$z = \frac{d}{\lambda} sin(\theta) \tag{15}$$

The sum on the right hand side of Equation 14 averages to zero except at:

$$z_m = \frac{\varphi_m}{2\pi} \tag{16}$$

Then Equation 14 becomes:

$$A(\omega, \theta(z_m), t) = \frac{1}{gN} \sum_{n=0}^{N-1} V_n(\omega, t) e^{j2\pi n\left(\frac{\phi_o}{2\pi} + \frac{m}{N}\right)} \tag{17}$$

The right hand side of Equation 17 is just the mth coefficient of the discrete Fourier transform of the amplitude of the N receivers. The beam width is given as:

$$\Delta z_m = \frac{1}{N} \tag{18}$$