Integration Time for 21cm Parabolic Cylinder Radio Telescope

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April 7, 2009

Received Power

From John Marriner's note "Digitized Response Function of a Phased Array of Antennae", the intensity of the radiation is related to the temperature. If there are many sources of radiation in the neighborhood of sky coordinates the intensities add linearly and the radiation can be characterized in terms of an intensity per steradian on the sky as follows:

$$I(\Omega) = 2\frac{k_B T_s(\Omega)}{\lambda^2} \tag{1}$$

The intensity I is measured in Watts/m^2/Hz/steradian. This can be taken as the definition of the temperature, but a blackbody radiator at temperature T_s would generate this flux per steradian per meter squared as seen from the receiving antenna. The intensity described in Equation 1 is for the sum of both modes of polarization. If the antenna is sensitive to only a single polarization mode, then the intensity available to the antenna:

$$I_{pol} = \frac{k_B T_s(\Omega)}{\lambda^2} \tag{2}$$

If the antenna has a solid beam angle of $\Delta\Omega_a$ steridians and is pointing at Ω_p, then the average power per area per bandwidth is:

$$\langle s_{av}(\Omega_p)\rangle = \frac{k_B\langle T_s(\Omega_p)\rangle}{\lambda^2}\Delta\Omega_a \tag{3}$$

The average amount of power per bandwidth received is proportional to the antenna area:

$$\langle p_{av}(\Omega_p)\rangle = \frac{k_B\langle T_s(\Omega_p)\rangle}{\lambda^2}\Delta\Omega_a g A_{em} \tag{4}$$

where A_{em} is the maximum available area of the antenna and g is the efficiency of the antenna (g<1). The antenna beam solid angle and maximum effective area are related:

$$\lambda^2 = A_{em}\Omega_a \tag{5}$$

The average amount of power per bandwidth received by the antenna is:

$$\langle p_{av}(\Omega_p)\rangle = g k_B\langle T_s(\Omega_p)\rangle \tag{6}$$

Assume that the antenna signal is sampled for τ_m seconds at a rate of r_s samples per second. The number of samples is:

$$N_s = \tau_m r_s \tag{7}$$

The frequency resolution of the measurement is:

$$\Delta f = \frac{1}{\tau_m} \tag{8}$$

The average power received in a bandwidth of Δf is:

$$\langle P(\Omega_p)\rangle = g k_B\langle T_s(\Omega_p)\rangle\Delta f \tag{9}$$

If the noise temperature of the antenna receiver is T_a, then the average total power measured is:

$$\langle P_T(\Omega_p)\rangle = k_B\big(g\langle T_s(\Omega_p)\rangle + T_a\big)\Delta f \tag{10}$$

Power Fluctuations

The amount of energy absorbed in the resolution bandwidth filter of the detector as a function of time is random. However, the average rate at which the energy is absorbed is constant. From these two preceding statements, the amount of energy absorbed in the detector in a measurement interval can be described by Poisson statistics where the mean and the standard deviation of the distribution are equal.

$$\langle \Delta P_T{}^2\rangle = \langle P_T\rangle^2 \tag{11}$$

For many measurements of the received power, the variance of the sample mean is given by the central limit theorem

$$\langle \Delta P_T{}^2\rangle_M = \frac{\langle \Delta P_T{}^2\rangle}{M} \tag{12}$$

where M is the number of measurements. Expressing the variance of the sample mean in terms of a temperature error:

$$\big(g k_B \Delta T_s(\Omega_p)\Delta f\big)^2 = \frac{1}{M}\Big(k_B\big(g\langle T_s(\Omega_p)\rangle + T_a\big)\Delta f\Big)^2 \tag{13}$$

Then the time it takes to make a measurement to this resolution is:

$$T_M = \frac{1}{\Delta f}\left[\frac{\langle T_s(\Omega_p)\rangle + \frac{1}{g}T_a}{\Delta T_s(\Omega_p)}\right]^2 \tag{14}$$

Meridian Telescope Configuration

Equation 14 assumes that the telescope tracks the pixel in the sky. For a parabolic meridian telescope of width W, the radiation pattern in right ascension is approximately:

$$\mathrm{R}(\phi) = \frac{\sin\left(\pi\frac{W}{\lambda}sin(\phi)\right)}{\pi\frac{W}{\lambda}sin(\phi)} \tag{15}$$

The resolution in right ascension of the antenna is:

$$\Delta\phi = sin^{-1}\left(\frac{\lambda}{W}\right) \tag{16}$$

If the meridian telescope does not track, then the sky pixel slice is in the antenna beam for:

$$\Delta t = 24hrs\frac{\Delta\phi}{2\pi} \tag{17}$$

The number of measurements per day on a given sky pixel slice is:

$$\mathrm{M_{day}} = 24hrs\frac{\Delta\phi}{2\pi}\Delta f \tag{18}$$

The average measurement rate for a given sky pixel slice is:

$$\langle \mathrm{r_M}\rangle = \frac{\Delta\phi}{2\pi}\Delta f \tag{19}$$

The time it takes to make as survey on a given sky pixel slice is:

$$\tau_{\text{survey}} = \frac{M}{\langle r_M \rangle} \tag{20}$$

or:

$$\tau_{\text{survey}} = \left[\frac{\langle T_s(\Omega_p)\rangle + \frac{1}{g}T_a}{\Delta T_s(\Omega_p)}\right]^2 \frac{2\pi}{\Delta\phi\Delta f} \tag{21}$$

or:

$$\tau_{\text{survey}} = \left[\frac{\langle T_s(\Omega_p)\rangle + \frac{1}{g}T_a}{\Delta T_s(\Omega_p)}\right]^2 \frac{2\pi}{sin^{-1}\left(\frac{\lambda}{W}\right)\Delta f} \tag{22}$$

Example

Figure 1 shows the survey time required for the following configuration

Amplifier Temperature	50K
Antenna Efficiency	80%
Resolution Bandwidth	3 MHz
Frequency	750 MHz
Antenna Width	12.5 m
Sky Noise	10 K

Table 1. Example Parameters for Figure 1.

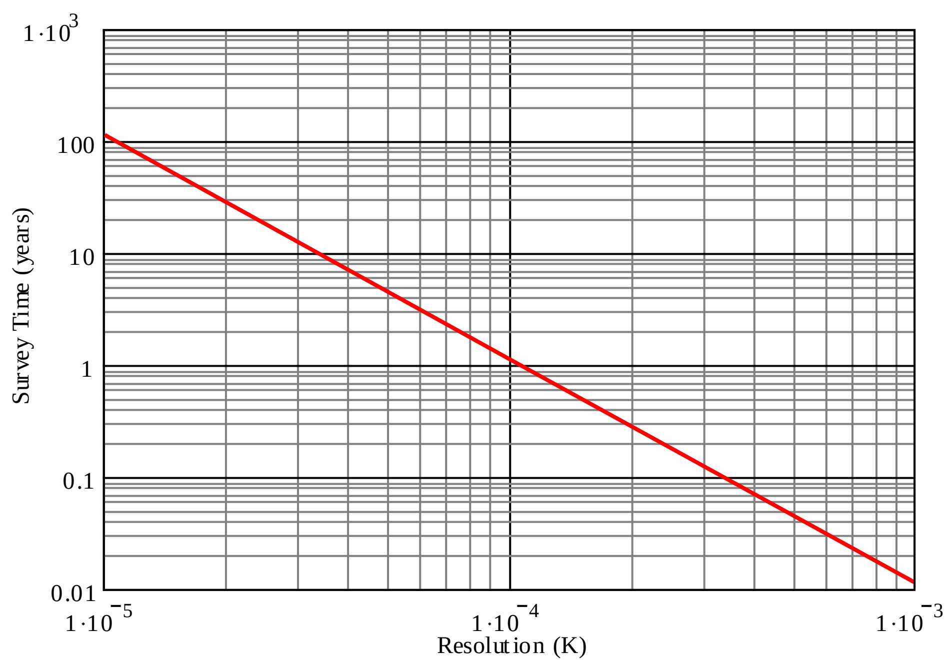

Figure 1. Survey time versus sensitivity