CRT Simulation

The simulation described here will create 3D pixel maps of observed flux, for various models of the sky, telescope, and electronics.

These data are used by analysis modules, prototypes of the ”final” scientific analysis.

We will use these results to optimize CRT design.

Chris Stoughton
John Marriner
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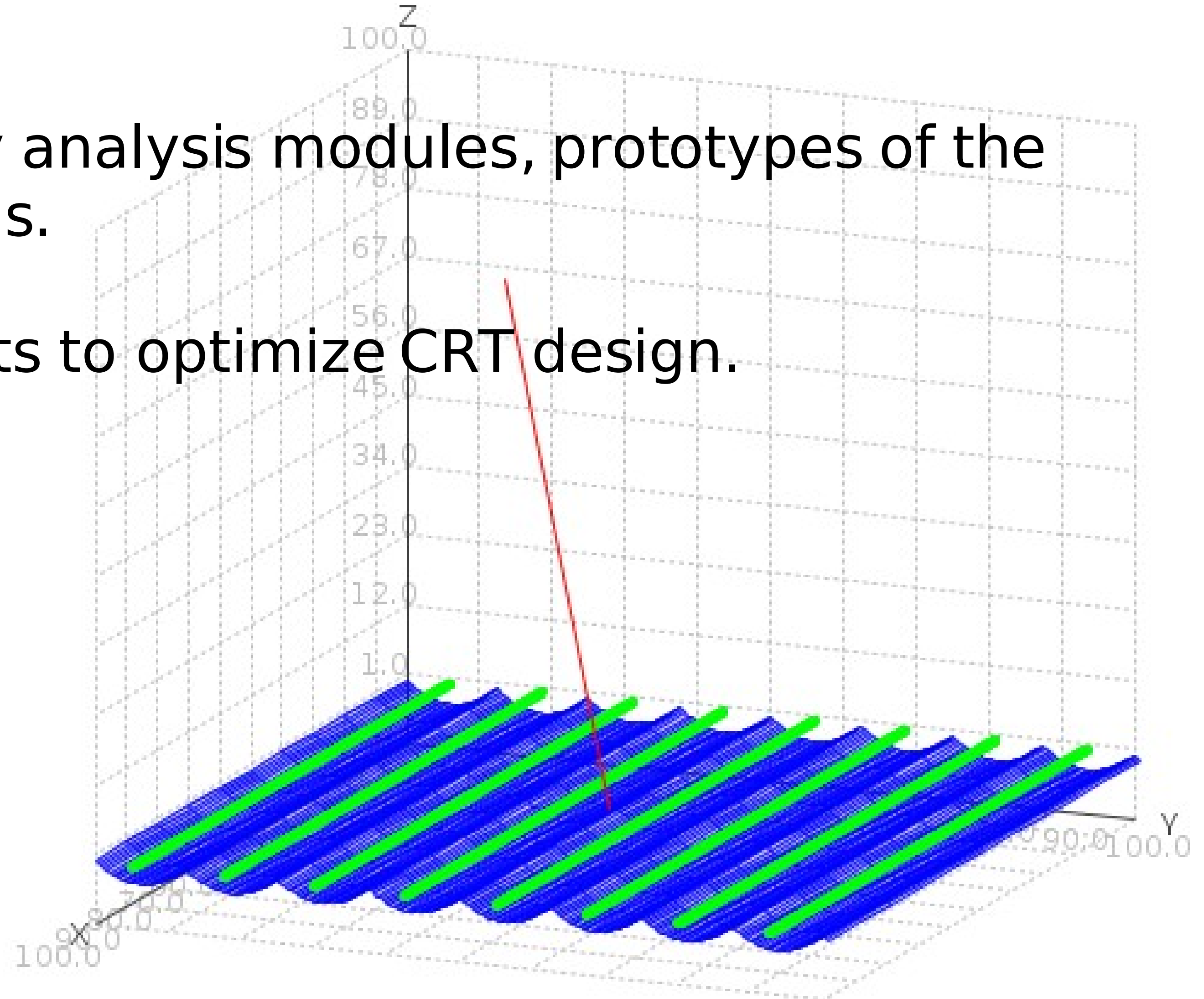


Purpose

- Measuring BAO signal is not trivial
- We need a detailed model of the sky and instrument:
  - Reasonable foregrounds and signal;
  - antenna and receiver models;
  - firmware options (zero padding, windowing, etc.)
- Begin with identical antennas and receivers, then evaluate impact of variations on the BAO analysis.

General Approach

- It is not feasible to simulate the time domain in detail – that would be 1 Tbyte/sec. Rather, we simulate the results of the DFTs.
- We will calculate the power spectrum: average power and rms fluctuations.
- The instrument is modeled as a linear transformation of power (T) on the sky.
- One ”exposure” yields 3D pixel map of T
    - coadd these to form the intensity map
    - this is input to the BAO analysis

Indices and Coordinates

Indices for the pixel map:

  υ: frequency, with $N_{\upsilon} = 2^{19}$

  m: angle from zenith parallel to cylinder (θ); $N_m = 2^{13}$

  n: rotation angle around cylinder axes (φ) $N_n = 2^6$

Indices for the detected frequencies and receivers:

  f: detected frequency bin (1D Fourier Transform of time)

  x: receiver number along a cylinder

  y: cylinder number for a receiver

Coordinates:

$\mathbf{k}=(k_x,k_y,k_z)$; $k=|\mathbf{k}|=\omega/c$

  with

$k_x = k\sin(\theta)$ $k_y = k\cos(\theta)\sin(\varphi)$ $k_z = k\cos(\theta)\cos(\varphi)$

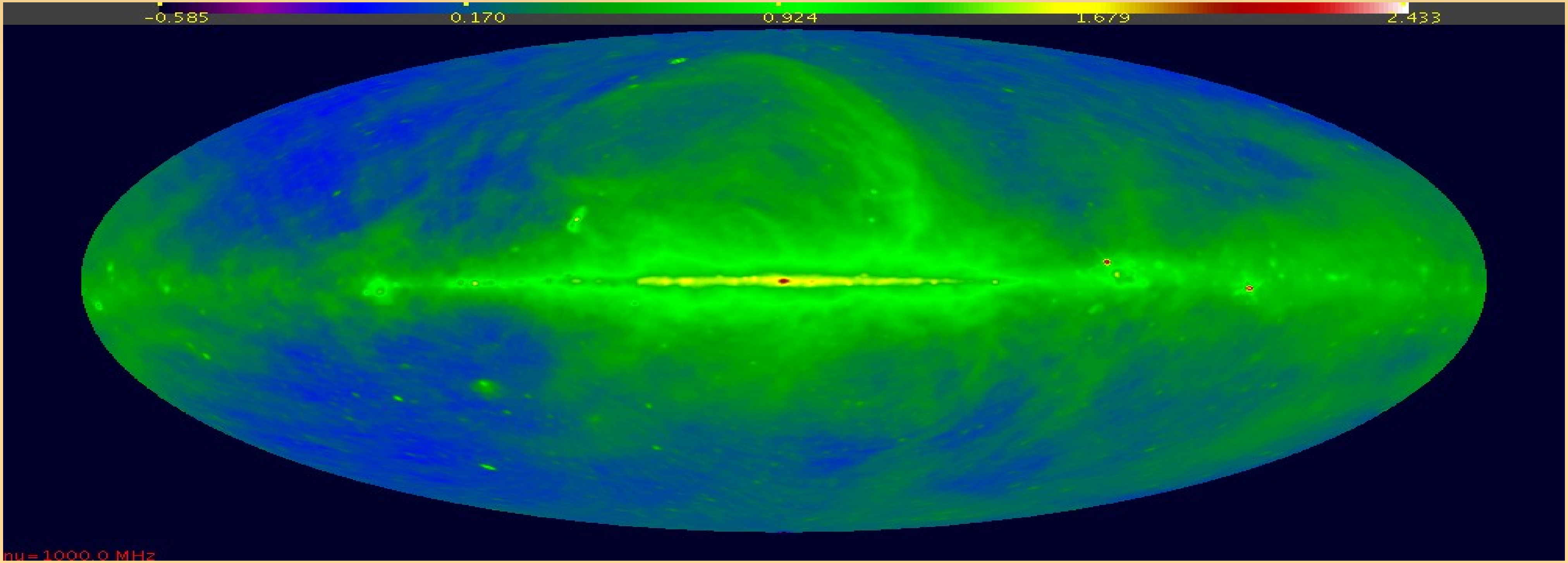
Define $\omega_f = 2\pi f/N_f\Delta t$, and angular size of pixels $\Delta\theta$, $\Delta\varphi$

Sky Model T(ra,dec,omega)

~~back~~foreground: Global Sky Model

http://space.mit.edu/home/angelica/gsm/

HI signal: N-body simulation, or discrete sources.



$$I(k) = \frac{2k_B T(f, \theta, \varphi)}{\lambda^2} \qquad \sigma(k) = \sqrt{I(k)}$$

Telescope Model

- Array of receivers with regular spacing
- $N_x = 2^{10}$ $D_x = 0.10$ m
- $N_y = 2^{8}$ $D_y = 12.5$ m
- Antenna model (simple, no polarization):

$$A_{xy}(\boldsymbol{k}) = \sqrt{d_x d_y}\,\frac{\sin(k_x d_x)}{k_x d_x}\,\frac{\sin(k_y d_y)}{k_y d_y}$$

Electronics Model

- Receiver bandwidth: 750 to 1000 Mhz (z from 0.4 to 0.9) constant gain: $G_{xy}(\omega)=1$
- Constant noise T 50 K: $\eta_{xy}(\omega)=\sqrt{k_B T_{Nxy}(\omega)}$
- Digitize voltage with $\Delta t=2$ ns for $N_t=2^{16}$ samples
- "exposure time" = 131 micro-sec.
- Front-end electronics; DFT to calculate 3D intensity pixel map.
- We simulate the results of the DFT for one exposure.

Simulate One Exposure

For each pixel in fmn (frequency, x-angle, y-angle) for the sky an instrument model, calculate mean power and its rms variation:

$$h_{fmn} = \sum_{\nu=0}^{N_\nu} \left[\left(\sum_{\mu=0}^{N_\mu} \sum_{\vartheta=0}^{N_\vartheta} \left| c_{fmn}\left(k_{\nu\mu\vartheta}\right) \right|^2 \right) + \left| d_{fmn}\left(\omega_\nu\right) \right|^2 \right] - P_f$$

$$\delta h_{fmn} = \sqrt{\sum_{\nu=0}^{N_\nu} \left[\left(\sum_{\mu=0}^{N_\mu} \sum_{\vartheta=0}^{N_\vartheta} \left| c_{fmn}\left(k_{\nu\mu\vartheta}\right) \right|^4 \right) + \left| d_{fmn}\left(\omega_\nu\right) \right|^4 \right] - \left(\delta P_f\right)^4}$$

For one exposure, in each pixel fmn, calculate h_{fmn} and δh_{fmn}.

Choose a random number from a normal distribution with this mean and sigma.

Don't forget to change the random number seed for the *next* exposure.

extra slides with equations

Calculate Mean Signal (1)

$$Z_{fxy}\left(\omega_\nu\right)=G_{xy}\left(\omega_\nu\right)\,e^{-i\omega_\nu t_0}\frac{e^{iN_f\left(\omega_f-\omega_\nu\right)\Delta t}}{1-e^{i\left(\omega_f-\omega_\nu\right)\Delta t}}\sqrt{\Delta f}$$

$$X_{fxy}\left(k_{\nu\mu\vartheta}\right)=A_{xy}\left(k_{\nu\mu\vartheta}\right)Z_{fxy}\left(\omega_\nu\right)\sqrt{\sin\theta_t\,\Delta\theta_t\,\Delta\varphi_t}$$

$$a_{fxy}\left(k_{\nu\mu\vartheta}\right)=X_{fxy}\left(k_{\nu\mu\vartheta}\right)\sigma\left(k_{\nu\mu\vartheta}\right)$$

$$c'_{fmy}\left(k_{\nu\mu\vartheta}\right)=\sum_{x=0}^{N_x}e^{-2\pi\mathrm{i}\,xm/N_x}e^{ixk_x d_x}a_{fxy}\left(k_{\nu\mu\vartheta}\right)$$

$$c_{fmn}\left(k_{\nu\mu\vartheta}\right)=\sum_{y=0}^{N_y}e^{-2\pi\mathrm{i}\,yn/N_y}e^{iyk_y d_y}c'_{fmy}\left(k_{\nu\mu\vartheta}\right)$$

Calculate Mean Signal (2)

$$b_{fxy}(\omega_\nu) = Z_{fxy}(\omega_\nu)\,\eta_{xy}(\omega_\nu)$$

$$d_{fmn}(\omega_\nu) = \sum_{y=0}^{N_x} \sum_{x=0}^{N_x} Y_{mn}^{xy}\, b_{fxy}(\omega_\nu)$$

$$p_{fxy} = \sum_{\nu=0}^{N_\nu} \left[\left(\sum_{\mu=0}^{N_\mu} \sum_{\vartheta=0}^{N_\vartheta} |a_{fxy}(k_{\nu\mu\vartheta})|^2 \right) + |b_{fxy}(\omega_\nu)|^2 \right]$$

$$P_f = \sum_{y=0}^{N_y} \sum_{x=0}^{N_x} p_{fxy}$$

$$h_{fmn} = \sum_{\nu=0}^{N_\nu} \left[\left(\sum_{\mu=0}^{N_\mu} \sum_{\vartheta=0}^{N_\vartheta} |c_{fmn}(k_{\nu\mu\vartheta})|^2 \right) + |d_{fmn}(\omega_\nu)|^2 \right] - P_f$$

Calculate Sigma of Signal

$$\delta p_{fxy} = \sqrt{\sum_{\nu=0}^{N_\nu} \left[\left(\sum_{\mu=0}^{N_\mu} \sum_{\vartheta=0}^{N_\vartheta} \left| a_{fxy}\left(k_{\nu\mu\vartheta}\right) \right|^4 \right) + \left| b_{fxy}\left(\omega_\nu\right) \right|^4 \right]}$$

$$\delta P_f = \sum_{y=0}^{N_y} \sum_{x=0}^{N_x} \left(\delta p_{fxy}\right)^2$$

$$\delta h_{fmn} = \sqrt{\sum_{\nu=0}^{N_\nu} \left[\left(\sum_{\mu=0}^{N_\mu} \sum_{\vartheta=0}^{N_\vartheta} \left| c_{fmn}\left(k_{\nu\mu\vartheta}\right) \right|^4 \right) + \left| d_{fmn}\left(\omega_\nu\right) \right|^4 \right] - \left(\delta P_f\right)^4}$$