Ray Tracing for an Offset Focus Parabolic Cylinder Antenna

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Introduction

The 21 cm project is based on a parabolic cylinder antenna with receivers placed periodically along the focus line of the antenna. It has been proposed that an offset focus parabolic cylinder antenna would eliminate multiple reflections between the feed line and the main cylinder. This note looks at the location of the focus as a function of the offset angle using simple ray tracing.

Parabola Equations

The equation of a parabola is given as:

$$z = f\left(\left(\frac{y}{2f}\right)^2 - 1\right) \quad (1)$$

where is the focal length of the parabola. The focus is located at the origin. This equation can be normalized by measuring position in units of twice the focal length:

$$v_p = \frac{1}{2}(u^2 - 1) \quad (2)$$

A ray incident on the parabola will have the form:

$$v_i = m_i u - v_{off} \quad (3)$$

where $\mathbf{m_i}$ is the slope of the ray which can be written as:

$$m_i = \tan\left(\frac{\pi}{2} + \theta\right) \quad (4)$$

where θ is the angle from vertical.
The slope of the parabola is given as:

$$m_p = \frac{dv_p}{du} = u \quad (5)$$

The incident ray will intersect the parabola at:

$$u_x = m_{px} = m_i + \sqrt{m_i^2 + 1 + 2v_{off}} \quad (6)$$

where $\mathbf{m_{px}}$ is the slope of the parabola at the intersection point. The slope of the normal vector is at the intersection point is given as:

$$m_{nx} = -\frac{1}{m_{px}} \quad (7)$$

The incident unit vector is:

$$\vec{a}_i = \frac{1}{\sqrt{m_i^2 + 1}}(\hat{u} + m_i\hat{v}) \quad (8)$$

The normal unit vector at the intersection point:

$$\vec{a}_{\perp} = \frac{1}{\sqrt{m_{px}^2 + 1}}\left(m_{px}\hat{u} - \hat{v}\right) \tag{9}$$

The angle between the incident ray and the normal ray is:

$$\cos(\theta_r) = \vec{a}_i \cdot \vec{a}_{\perp} = \frac{m_{px} - m_i}{\sqrt{m_i^2 + 1}\sqrt{m_{px}^2 + 1}} \tag{10}$$

The slope of the reflected ray is:

$$m_r = \tan\left(\frac{\pi}{2} + \theta - 2\theta_r\right) \tag{11}$$

The equation for the reflected ray is:

$$v_r = m_r(u - u_x) + m_i(u_x) \tag{12}$$

Plots

The plots are in units of focal length. The maximum width of the parabola drawn is four times the focal length and the maximum height is equal to the focal length. The incident rays are dashed and the reflected rays are solid. The ray colored in red has a reflected ray that is horizontal and the ray colored in black is the ray that is reflected directly back on the incident ray.

Shifting the focus off center works if the ratio of the focal length to width (f-ratio) is large. The ray-tracing shows that for small f-ratios, there is no clearly defined focus for an offset focus. The larger the offset, the worse the focusing becomes.

To make an off-axis parabola, a large f-ratio is needed which would have the towers holding the pre-amp very high up in the air. The only way to make an offset-fed telescope with a small f-ratio so that the feed-tower is not way up in the air is to feed it on-axis and cut away part of the parabola.

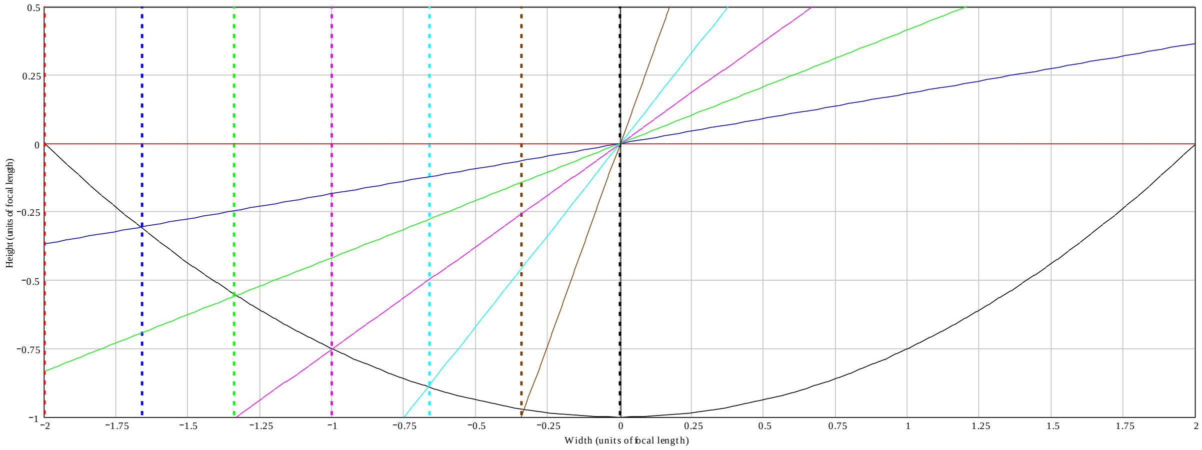

Figure 1. Incident ray is zero degrees from vertical.

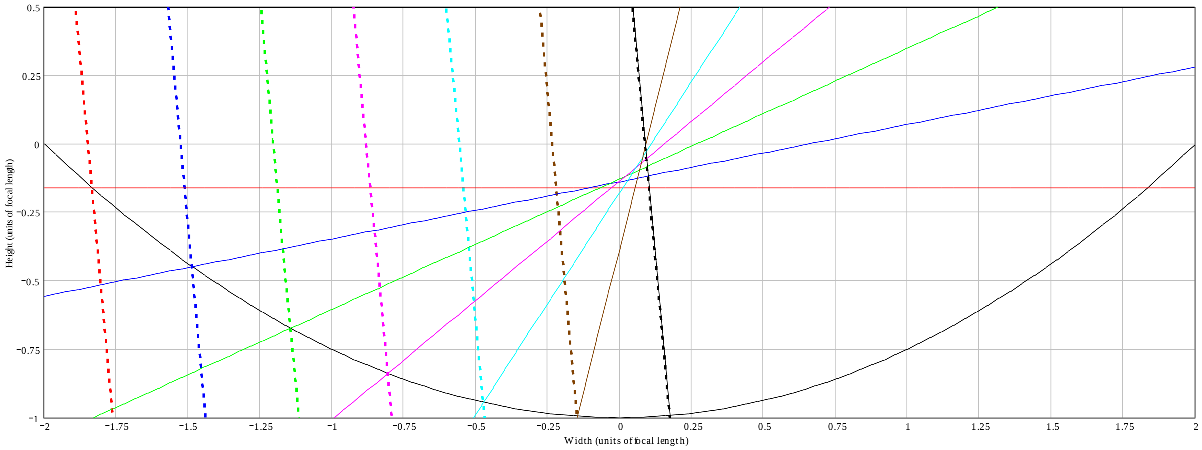

Figure 2. Incident ray is five degrees from vertical.

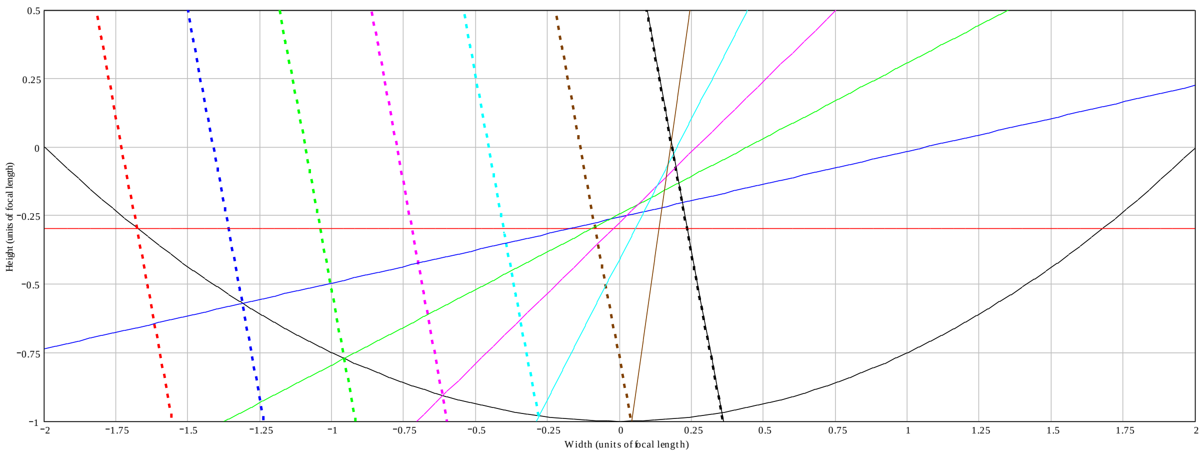

Figure 3. Incident ray is ten degrees from vertical.

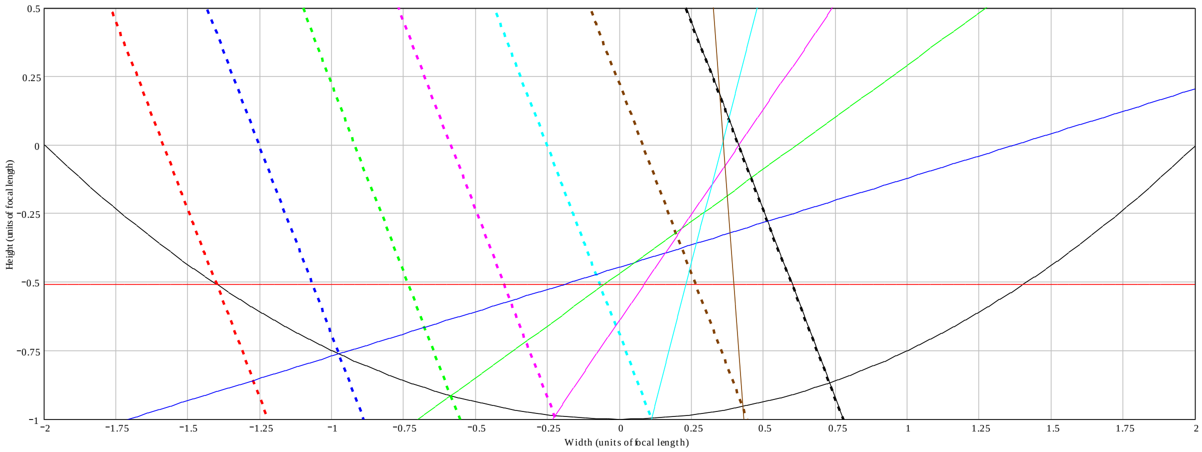

Figure 4. Incident ray is twenty degrees from vertical.

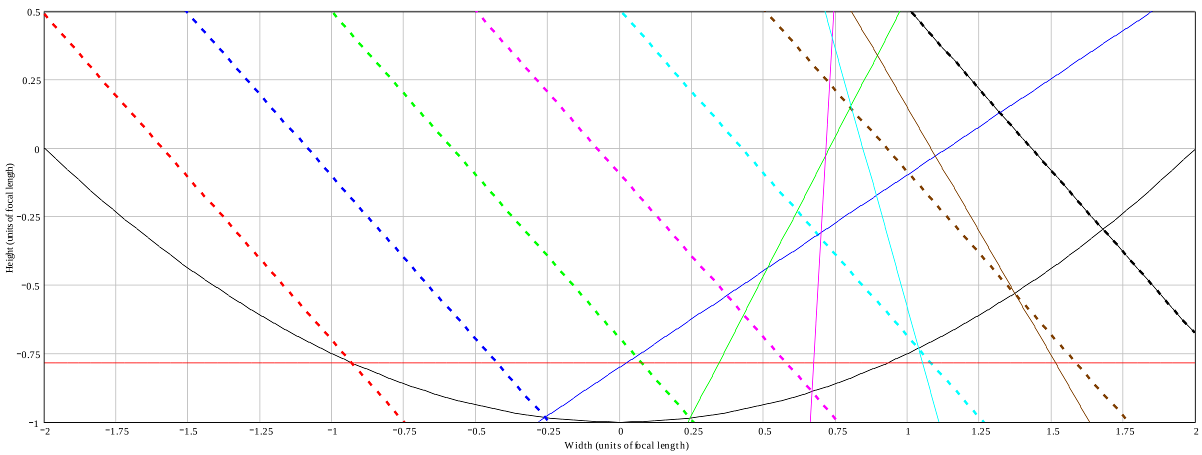

Figure 5. Incident ray is forty degrees from vertical.