Integration Time for a Parabolic Dish Radio Telescope

Dave McGinnis
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Resolution

Effective area of a parabolic dish:

$$A_{em} = \pi \left(\frac{D}{2}\right)^2 \tag{1}$$

where D is the diameter. In general, the beam area of the antenna is:

$$\Omega_A = \frac{\lambda^2}{A_{em}} \tag{2}$$

For a dish:

$$\Omega_A = \frac{1}{\pi}\left(\frac{2\lambda}{D}\right)^2 \tag{3}$$

The beam width $\Delta\theta$ is given as:

$$\Omega_A = \pi \left(\frac{\Delta\theta}{2}\right)^2 \tag{4}$$

or:

$$\Delta\theta = \frac{4\lambda}{\pi D} \tag{5}$$

In general, the directivity or gain of the antenna is given as:

$$G = \frac{4\pi}{\Omega_A} \tag{6}$$

For a parabolic dish:

$$G = \left(\frac{\pi D}{\lambda}\right)^2 \tag{7}$$

<u>Some Numbers:</u>

f	=	1	GHz			
λ	=	0.3	m			
Δθ	=	5	degrees	=	87.27	mrad
D	=	4.3	meters	=	14.36	feet
G	=	2028		=	33 dB	

Integration Time

Assume that object drifts across beam of antenna and signal is integrated along the drift.

$$\omega_d T_{int} = x_{int}\Delta\theta \tag{8}$$

Where x_{int} is some duty factor of the integration and ω_d is given as:

$$\omega_d = \frac{2\pi}{24hr \cdot 3600\frac{sec}{hr}} = 72.72\mu rad/sec \tag{9}$$

From the note: "Integration length for 21cm", D. McGinnis, ver 3. (Project Document 290-v3), the integration length needed for an accuracy **a** of an object temperature T_s measured over a resolution bandwidth Δf is:

$$T_{int} \approx \frac{1}{\Delta f}\left(\frac{1}{g}\frac{T_A}{aT_s}\right)^2 \tag{10}$$

for $T_A >> T_s$, where T_A is the amplifier temperature, and g is the efficiency of the antenna feed ($0<g<1$). Using Equations 5, 8, and 10:

$$aT_s = T_A\sqrt{\frac{1}{x_{int}g^2}\frac{\pi D}{4\lambda}\frac{\omega_d}{\Delta f}} \tag{11}$$

<u>Some Numbers:</u>

f	=	1	GHz			
λ	=	0.3	m			
Δθ	=	5	degrees	=	87.27	mrad
D	=	4.3	meters	=	14.36	feet
g	=	0.5				
x_{int}	=	0.5				
Δf	=	100	kHz			
N_f	=	1	dB	Amplifier Noise Figure		
T_a	=	76	degrees Kelvin			
aT_s	=	$19\text{x}10^{-3}$ degrees Kelvin				