Derivation of Formula for Liquid Argon Flashing to Vapor When Pressure Is Reduced

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Apply equations on page 118 for a uniform state process from Fundamentals of Classical Thermodynamics, Van Wylen and Sonntag, 1965, to a liquid argon dewar that undergoes a sudden reduction in pressure. Assume no external heat load on the dewar and thermodynamic equilibrium before and after the loss of pressure. Assume the control volume, over which the mass and energy balance equations are applied is the dewar inner vessel. The dewar initially is 100% saturated liquid at a relatively high pressure.

State 1 is when the dewar is full of saturated liquid argon at a pressure aove atmopheric. V is the volume of the dewar. The mass at state 1 is:

$$M_1 = \rho_{f1} V$$

In state 2, the dewar is at atmosphic pressure, or some pressure lower than the pressure at state 1. The argon is a two phase mixture of vapor and liquid. Define the variable y as the volume fraction of the dewar that contains argon vapor. The volume fraction of the liquid would then be the quantity 1-y. The mass of Argon liquid in the dewar in state 2 is:

$$M_{f2} = \rho_{f2} (1 - y) V$$

Mass of Argon vapor in the dewar in state 2 is:

$$M_{g2} = \rho_{g2}\, y\, V$$

mass of vapor that vents from the dewar, when the dewar pressure is reduced is:

$$M_e = M_1 - M_{f2} - M_{g2}$$

Apply conservation of energy eqation to the dewar for this process. The internal energy in the dewar in state 1 equals the internal energy in state 2 plus th enthalpy of the departing vapor. The enthalpy of the departing vapor would be the enthalpy of saturated argon vapor at the average pressure between states 1 and 2.

$$M_1\, u_{f1} = M_{f2}\, u_{f2} + M_{g2}\, u_{g2} + M_e\, h_{eg}$$

Substitute in the mass expressions.

$$(\rho_{f1} V)\, u_{f1} = (\rho_{f2} (1 - y) V)\, u_{f2} + (\rho_{g2}\, y\, V)\, u_{g2} + (\rho_{f1} V - \rho_{f2} (1 - y) V - \rho_{g2}\, y\, V)\, h_{eg}$$

Rearrange the equation

$$\rho_{f2}\, V\, u_{f2} - \rho_{f2}\, y\, V\, u_{f2} + \rho_{g2}\, y\, V\, u_{g2} + \rho_{f1}\, V\, h_{eg} - \rho_{f2}\, V\, h_{eg} + \rho_{f2}\, y\, V\, h_{eg} - \rho_{g2}\, y\, V\, h_{eg} = \rho_{f1}\, V\, u_{f1}$$

Divide through by V and put terms without y on left hand side of equation.

$$-\rho_{f2}\, y\, u_{f2} + \rho_{g2}\, y\, u_{g2} + \rho_{f2}\, y\, h_{eg} - \rho_{g2}\, y\, h_{eg} = \rho_{f1}\, u_{f1} - \rho_{f2}\, u_{f2} - \rho_{f1}\, h_{eg} + \rho_{f2}\, h_{eg}$$

Factor y out of the left hand side of the equation and solve for it.

$$y\,(-\rho_{f2}\, u_{f2} + \rho_{g2}\, u_{g2} + \rho_{f2}\, h_{eg} - \rho_{g2}\, h_{eg}) = \rho_{f1}\, u_{f1} - \rho_{f2}\, u_{f2} - \rho_{f1}\, h_{eg} + \rho_{f2}\, h_{eg}$$

$$\frac{y\,(-\rho_{f2}\, u_{f2} + \rho_{g2}\, u_{g2} + \rho_{f2}\, h_{eg} - \rho_{g2}\, h_{eg})}{-\rho_{f2}\, u_{f2} + \rho_{g2}\, u_{g2} + \rho_{f2}\, h_{eg} - \rho_{g2}\, h_{eg}} = \frac{\rho_{f1}\, u_{f1} - \rho_{f2}\, u_{f2} - \rho_{f1}\, h_{eg} + \rho_{f2}\, h_{eg}}{-\rho_{f2}\, u_{f2} + \rho_{g2}\, u_{g2} + \rho_{f2}\, h_{eg} - \rho_{g2}\, h_{eg}}$$

Cancel out common terms

$$y = \frac{\rho_{f1}\, u_{f1} - \rho_{f2}\, u_{f2} - \rho_{f1}\, h_{eg} + \rho_{f2}\, h_{eg}}{-\rho_{f2}\, u_{f2} + \rho_{g2}\, u_{g2} + \rho_{f2}\, h_{eg} - \rho_{g2}\, h_{eg}}$$

Rearrange terms.

$$y = \frac{\rho_{f1}\, u_{f1} - \rho_{f2}\, u_{f2} - \rho_{f1}\, h_{eg} + \rho_{f2}\, h_{eg}}{\rho_{g2}\, u_{g2} - \rho_{f2}\, u_{f2} - \rho_{g2}\, h_{eg} + \rho_{f2}\, h_{eg}}$$