TURTLE, G4beamline, Rectangular Bends, and Fringe Fields

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Overview

- This is a comparison of the methods used by TURTLE and G4beamline.
- It is not an attempt to defend the physics model of either.

TURTLE

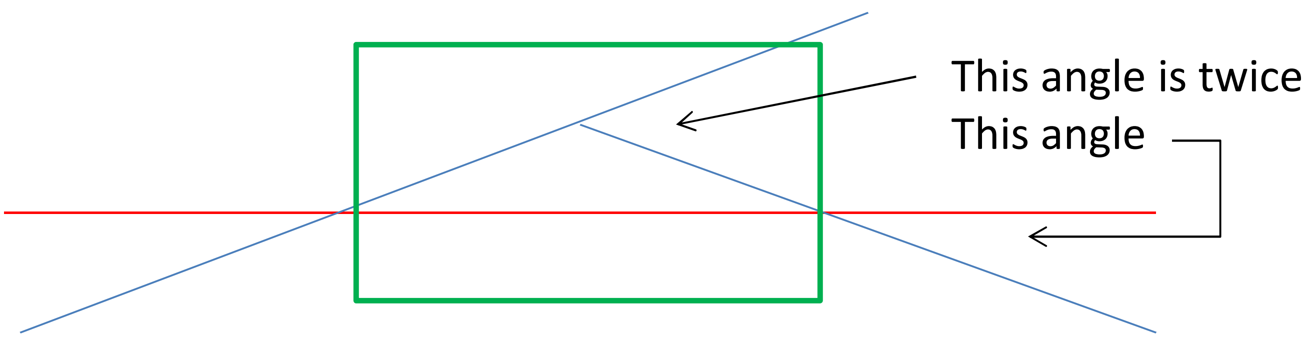
- “Trace Unlimited Rays Through Lumped Elements”
- Uses matrix optics, up to third-order, to trace individual rays through an optics system.
- Refer to D.C. Carey, et al., “TURTLE with MAD Input”, SLAC-R-544/Fermilab-Pub-99/232

G4Beamline

- “G4Beamline is a particle tracking and simulation program based on the Geant4 toolkit …”
- Tom Roberts, “G4beamline User’s Guide”

Rectangular Bend
"What It Is"

- Bending magnet is a rectangle – although it can be curved, as long as …
- Angle between central ray and a line normal to face of magnet is one-half bend angle.



Rectangular Bend
Difference in Models

TURTLE:

- At this point, fringe fields are ignored.
- TURTLE treats a rectangular bend as a sector bend with rotated pole faces. The pole-face rotation leads to a focusing term – defocusing in the bend plane, focusing in the non-bend plane.
- The pole-face rotation is treated as a hard-edge, i.e., the field ends abruptly at the end of the magnet.
- A new coordinate is introduced which extends perpendicular to the (sector) pole face and is defined through the rotated face.
- This gives rise to the defocusing term in the bend plane – which exactly cancels the focusing term a sector bend would have.
- The "extra" field, along with Maxwell's equations, give rise to a term in the non-bend plane. Integrating through this results in a focusing term equal to the (bend plane) focusing term.
- For details, refer to David C. Carey, "The Optics of Charged Particle Beams".

Rectangular Bends
Difference in Models

G4beamline

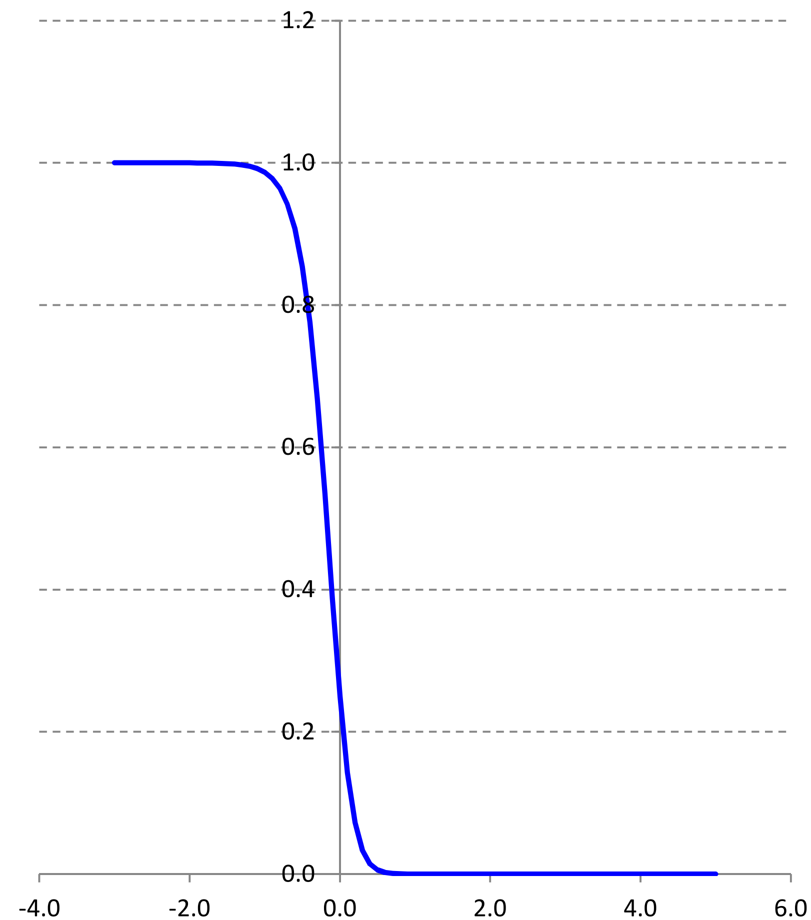
- At this point, fringe fields are ignored.
- Treats a rectangular bend as a magnetic field perpendicular to bend plane.
- Due to geometry of a uniform field in a rectangular region, there is no bend-plane focusing.
- Because there is no bend-plane magnetic field component, there no non-bend-plane focusing.
- $\vec{v} \times \vec{B} = v_z * B_y = F_x$

Rectangular Bends Summary

- A “rectangular bend without fringe fields” is modeled differently in TURTLE and G4beamline.
- This is more than a statement about the mathematics (linear algebra versus numeric integration).
- The *physics model* differs.
- Or, as Doug observed, $0 \neq 0$!

Fringe Field
“What It Is”

- The “fringe field” is the magnetic field that “sticks out of the magnet”.
- However, it also “sticks into the magnet”, that is, component perpendicular to the bend plane begins to decrease before the edge of the pole tip.



Fringe Field
TURTLE

- Assume an effective boundary defined by:

$$\int_{-\sigma_1}^{\infty} B_y(\sigma) = \sigma_1 B_0$$

"σ" is distance perpendicular to pole face.

That is, the integrated field beyond the effective boundary equals the missing integrated field inside the boundary, up to some point.

- Lots of math, find that the pole face is effectively rotated by an additional amount

$$\varphi = \frac{g}{\rho}\frac{1 + sin^2\beta}{cos\beta} I_2$$

Where g= gap, ρ=radius of curvature, β=one-half bend angle.

- What is I_2?

Fringe Field
TURTLE

- $I_2 = \int_{-\sigma_1}^{\infty} \frac{B(\sigma)_y\left(B_0 - B(\sigma)_y\right)}{g{B_0}^2} d\sigma$

- Well, that's real clear …

Fringe Fields G4beamline

- Assumes magnetic field satisfies Maxwell's equations to first order, and the field perpendicular to the bend plane can be characterized by

$$B_y(z) = F(z)B_0$$

And

$$F(z) = \frac{1}{1 + e^{\sum_{i=0}^{n} a_i \left(\frac{z}{g}\right)^i}}$$

- This form should be used for $-3 \leq \frac{z}{g} \leq 5$
- Let $\sigma = \frac{z}{g}$ and $F(z) \rightarrow F(\sigma)$
- Note: *this σ is <u>not</u> the σ in* TURTLE.

Fringe Fields
Tying it Together

- Now, we can re-write the equation for I_2 (TURTLE) in terms of F (G4beamline).

$$I_2 = \int_{\sigma_1}^{\sigma_2} \frac{F(\sigma)B_0(B_0 - F(\sigma)B_0)}{g{B_0}^2}\,d\sigma$$

$$= \frac{(z_2 - z_1)}{g}\left[\int_{\sigma_1}^{\sigma_2} F(\sigma)\,d\sigma - \int_{\sigma_1}^{\sigma_2} F^2(\sigma)\,d\sigma\right]$$

- We have a way to compare TURTLE to G4beamline if we know F.

Conclusions (Problems)

- Need to measure or simulate field to determine F.
- Our magnets typically have $g = 180mm$ and $L = 800mm$, so $^{L}/_{g} \approx 4.5$, so integrating three gap-widths from either side may be problematic (depends on how “flat” field is in central region).