Geometrical Model of Illumination of Parabolic Cylinder

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March 12, 2011

Introduction

For the case of an isotropic feed, a short section of cylinder can be modeled as a rectangular aperture. This is because the electromagnetic path length from the reflecting surface to the focal point is the same for all points on the cylinder surface and the feed pattern is isotropic. In the case in which the feed pattern is not isotropic (i.e. the feed dimensions are a significant fraction of a wavelength), the aperture equations can be modified to give an approximation of the cylinder beam pattern. These modifications can lend to an estimate of the fraction of the surface that is illuminated by the feed pattern.

Collecting Area of an Aperture

Consider an aperture in a plane. If all the energy incident on the aperture is to be collected at a single feed point, the antenna pattern of the aperture is given as:

$$F(\Omega) = \frac{1}{A}\iint_{A'} e^{-j\vec{\beta}(\Omega)\cdot\vec{r}'} dA' \tag{1}$$

where A is the area of the aperture, Ω is the angle of the incoming radiation and β is the incoming wave vector in which:

$$|\beta| = \frac{2\pi}{\lambda} \tag{2}$$

where λ is the wavelength of the radiation. The beam area of the beam is given as:

$$\Omega_B = \iint |F(\Omega)|^2 d\Omega \tag{3}$$

The directivity or gain of the aperture is given as:

$$D = \frac{4\pi}{\Omega_B} \tag{4}$$

The effective collecting area of the aperture is:

$$A_e = \frac{\lambda^2}{\Omega_B} \tag{5}$$

Consider a rectangular aperture in which the length of the aperture in the x direction is ΔL and the width of the aperture in the y direction is W. Given the following spherical coordinate system:

$$x = sin(\theta) \tag{6}$$

$$y = cos(\theta)sin(\phi) \tag{7}$$

$$z = cos(\theta)cos(\phi) \tag{8}$$

The wave vector is given as:

$$\vec{\beta}(\Omega(\theta,\phi)) = \frac{2\pi}{\lambda}(sin(\theta)\hat{x} + cos(\theta)sin(\phi)\hat{y}) \tag{9}$$

Equation 1 becomes:

$$F(\theta,\phi) = \left(\frac{1}{\Delta\mathrm{L}}\int_{-\frac{\Delta\mathrm{L}}{2}}^{\frac{\Delta\mathrm{L}}{2}} e^{-j\frac{2\pi}{\lambda}sin(\theta)x}dx\right)\left(\frac{1}{\mathrm{W}}\int_{-\frac{\mathrm{W}}{2}}^{\frac{\mathrm{W}}{2}} e^{-j\frac{2\pi}{\lambda}cos(\theta)sin(\phi)y}dy\right) \qquad (10)$$

The antenna pattern is:

$$\mathrm{F}(\theta,\phi) = \frac{sin\left(\pi\frac{\Delta\mathrm{L}}{\lambda}sin(\theta)\right)}{\pi\frac{\Delta\mathrm{L}}{\lambda}sin(\theta)}\frac{sin\left(\pi\frac{W}{\lambda}cos(\theta)sin(\phi)\right)}{\pi\frac{W}{\lambda}cos(\theta)sin(\phi)} \qquad (11)$$

The effective collection area of the aperture equals the physical collection area of the aperture.

$$\mathrm{A_e} = W\Delta\mathrm{L} \qquad (12)$$

In the limit of ΔL ->0, then

$$\mathrm{F}(\theta,\phi) = \frac{sin\left(\pi\frac{W}{\lambda}cos(\theta)sin(\phi)\right)}{\pi\frac{W}{\lambda}cos(\theta)sin(\phi)} \qquad (13)$$

Feed Pattern

Consider the illumination of a parabolic cylinder by a feed element placed at its focus. The equation for a parabola is:

$$z = \frac{1}{4f}y^2 - f \qquad (14)$$

The equation for a ray coming from the feed is:

$$z = \frac{y}{tan(\psi)} \qquad (15)$$

where ψ is the angle of the ray away from the vertical. The intersection of the ray and the parabola is at:

$$\frac{y}{W} = 2\frac{f}{W}tan\left(\frac{\psi}{2}\right) \qquad (16)$$

Or

$$\psi = 2tan^{-1}\left(\frac{W}{f}\frac{2y}{W}\right) \qquad (17)$$

Now assume that the feed has a pattern:

$$\mathrm{F_{feed}}(\theta,\phi) = \mathrm{a}\big(sin(\theta)\big)\mathrm{b}\big(cos(\theta)sin(\psi)\big) \qquad (18)$$

Equation 10 can be modified to:

$$F(\theta,\phi) = \mathrm{a}\big(sin(\theta)\big)\left(\frac{\int_{-\frac{W}{2}}^{\frac{W}{2}} \mathrm{b}\left(cos(\theta)sin\left(2tan^{-1}\left(\frac{W}{f}\frac{2y}{W}\right)\right)\right)e^{-j\frac{2\pi}{\lambda}cos(\theta)sin(\phi)y}dy}{\int_{-\frac{W}{2}}^{\frac{W}{2}} b\left(cos(\theta)sin\left(2tan^{-1}\left(\frac{W}{f}\frac{2y}{W}\right)\right)\right)dy}\right) \quad (19)$$

Using Equation 3, an estimate of the width of the beam angle in the φ direction is given as:

$$\Delta\phi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |F(0,\phi)|^2 d\phi \quad (20)$$

From Equation 5, and estimate of the effective illumination of the cylinder is:

$$\frac{\mathrm{W_e}}{\mathrm{W}} = \frac{\lambda}{\Delta\phi} \quad (21)$$

Example: Gaussian Antenna Feed Pattern on a Cylinder

It has been suggested the antenna pattern of a four square antenna can be model with a gaussian field pattern. Consider a gaussian feed pattern of the form:

$$b(\psi) = e^{-\frac{1}{2}\ln(10)\left(\frac{2\psi}{\Delta\psi_{10\mathrm{dB}}}\right)^2} \quad (22)$$

where $\Delta\psi_{10dB}$ is the full width of the power beam pattern 10dB down from the maximum. Note that the factor of ½ in front of exponent terms is because the power pattern is the magnitude squared of the field pattern b(ψ). Figure 1 shows the effective width as a function of focal length over width ratio for a 20 meter cylinder.

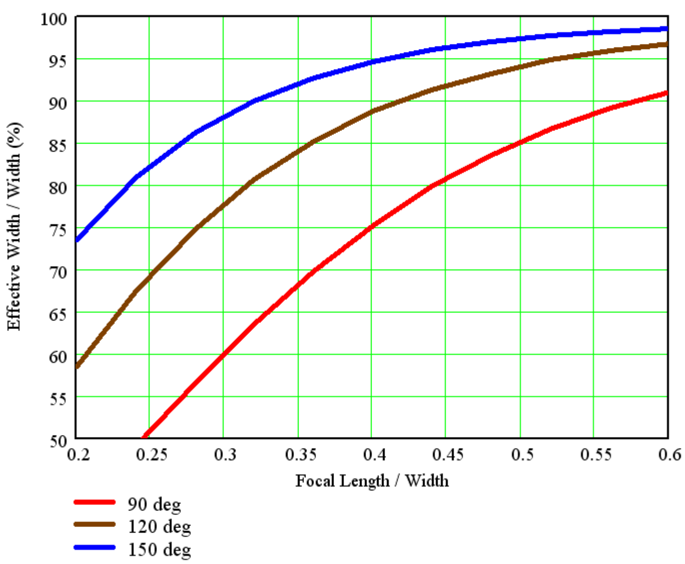

Figure 1. Effective illumination width of a cylinder versus f ratio for various gaussian feed patterns. The feed pattern for the red trace has a 10dB power full-width of 90 degrees. The brown trace has a power width of 120 degrees and the blue trace of 150 degrees.