

# 21cm Cylindrical Radio Telescope Design and Simulation

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Fermilab

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# MOTIVATION

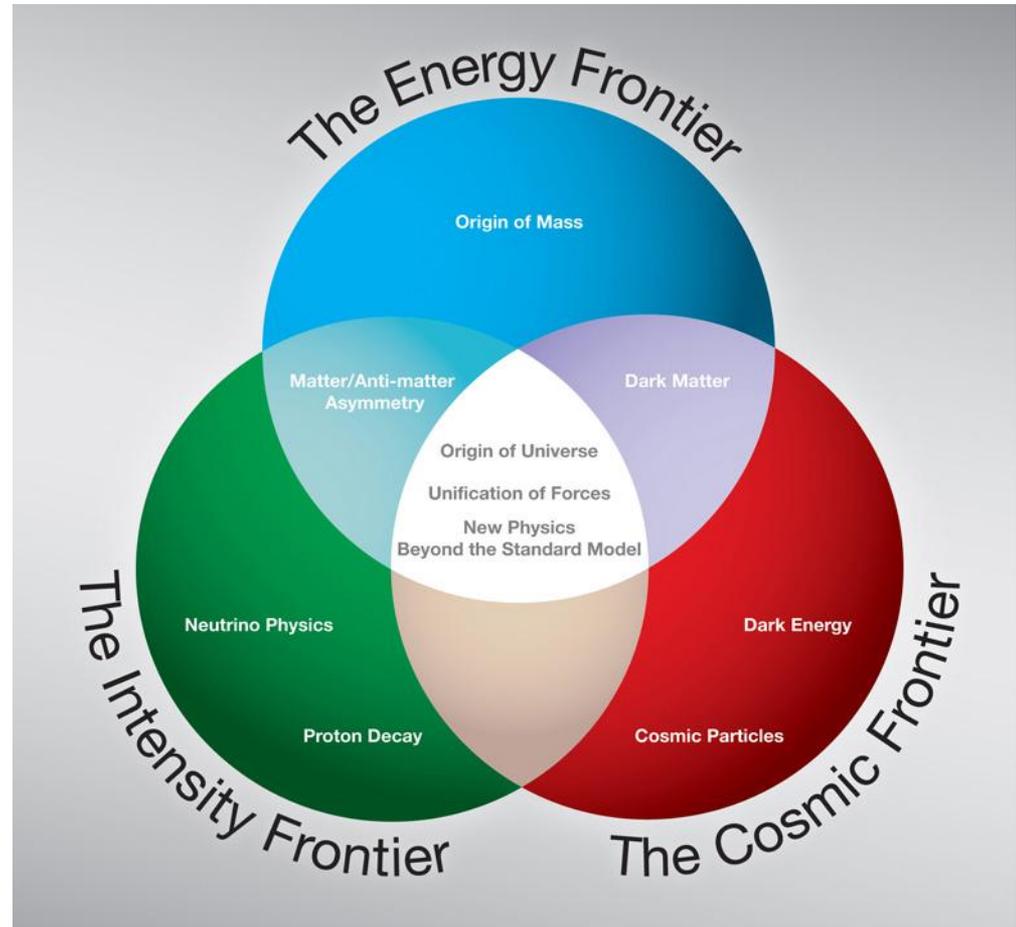
# Fermilab

- Fermilab is an United States national laboratory
  - located ~60 km southwest of Chicago
  - Occupying ~27 square kilometers of land
  - Employing ~2000 people



# Fermilab's Mission

- The fundamental mission of Fermilab is to study
  - High-Energy Physics,
  - the science of matter,
  - space and time
- Fermilab's mission is divided into three “frontiers”
  - The energy frontier
    - 2 TeV Tevatron
    - 7 TeV LHC
  - The intensity frontier
    - 325 kW (700kW) 120 GeV Main injector
  - The cosmic frontier
    - Center for Particle Astrophysics



# Fermilab Center for Particle Astrophysics (FCPA)

- The center consists of about 70 people
  - 60 % scientists
  - 25% staff
  - 15% postdocs
- The center is involved in the following projects
  - Chicagoland Observatory for Underground Particle Physics (COUPP)
  - Cryogenic Dark Matter Search (CDMS)
  - Dark Energy Survey (DES)
  - GammeV
  - Pierre Auger Observatory
  - Sloan Digital Sky Survey (SDSS)
  - Joint Dark Energy Mission (JDEM formerly known as SNAP)
  - Theoretical Astrophysics (~15 physicists)
  - Experimental Astrophysics

# Fermilab and 21cm

- In 2007, Jeff Peterson from Carnegie-Mellon University gave a talk at the to FCPA on “The Hubble Sphere Hydrogen Survey”
  - Which describes intensity mapping of the 21cm line
  - And describes the concept of an FFT telescope with cylindrical reflectors
- A few of us at Fermilab became intrigued with the concept because
  - We are very interested in dark energy
  - We would like to get in on the “ground floor” on 21cm intensity mapping
  - The project overlays nicely with our expertise
    - Large area sky surveys
    - Large volume data acquisition
    - RF technology
    - Digital signal processing
    - Management of medium to large scale projects
- Fermilab joined the 21cm Cylindrical Radio Telescope (CRT) collaboration

# THE 21CM CRT DESIGN

# Design Process

- Define the science
  - Dark energy
- Define parameter that measures success
  - Dark Energy Task force Figure of Merit
- Define science technique
  - Baryon Acoustic Oscillations with intensity mapping
- Pick an Instrument
  - Develop a rough engineering model
  - Estimate the cost versus science of the instrument
  - Pick a parameter set or “punt”

# Science Technique

- Measure Baryon Acoustic Oscillations at large red-shifts with intensity mapping
- To peer deep into large red-shifts, we use a hydrogen hyperfine transition at 1.42 GHz to make a 3-D radio intensity map of the universe
- By intensity map, we mean that:
  - galaxies are not spatially resolved
  - the 21cm line is not resolved in frequency
- With the 3-D radio intensity map, we will pull the unique BAO signal out of a map that is dominated by foregrounds.
  - Foregrounds consist of galactic synchrotron emission, point radio sources, etc.
  - Foreground subtraction will be the most difficult part of the project

# **INSTRUMENT CHOICE**

# FFT Radio Telescopes

- 3-D sky surveys require
  - Large collecting area
  - Good resolution
  - Large frequency bandwidth
  - High speed
- For a given sensitivity, the survey speed is proportional to the number of electronic channels.
  - To do a Stage 3-4 Dark Energy experiment using BAO in 2-3 years, ~2000 channels are needed
- We think the best fit for these requirements is a FFT Radio Telescope<sup>1</sup>
- An FFT Radio Telescope is composed of:
  - arrays of low gain, wide beam width, antennae
  - connected to low-noise, high speed, electronics.

<sup>1</sup>Omniscopes: Large Area Telescope Arrays with only  $N \log N$  Computational Cost, M. Tegmark - <http://arxiv.org/abs/0909.0001v1>

# Visibilities

- A standard radio interferometer measures information(visibility) from the cross correlation of 2 receivers as a function of the distance between receivers.
  - For an array of  $N$  receivers, **there are  $N(N-1)/2$**  possible products to compute.
    - For  $N=2000$ , there are  $\sim 2 \times 10^6$  visibilities
- For an FFT Radio Telescope
  - Receivers are located uniformly in an array
  - $N$  electronic beams are formed on the sky simultaneously by computing the spatial Fourier transform of the receivers' voltages.
  - The power spectrum of each electronic beam contains all the possible visibilities.
  - The computational load goes as  **$N \log N$**
  - But because of the uniform spacing required for spatial Fourier Transform, there are many redundant baselines.
  - However, these redundant baselines provide:
    - Better signal to noise (for quick survey speed)
    - Flexibility for calibration or insensitivity to calibration errors.

# New Technology

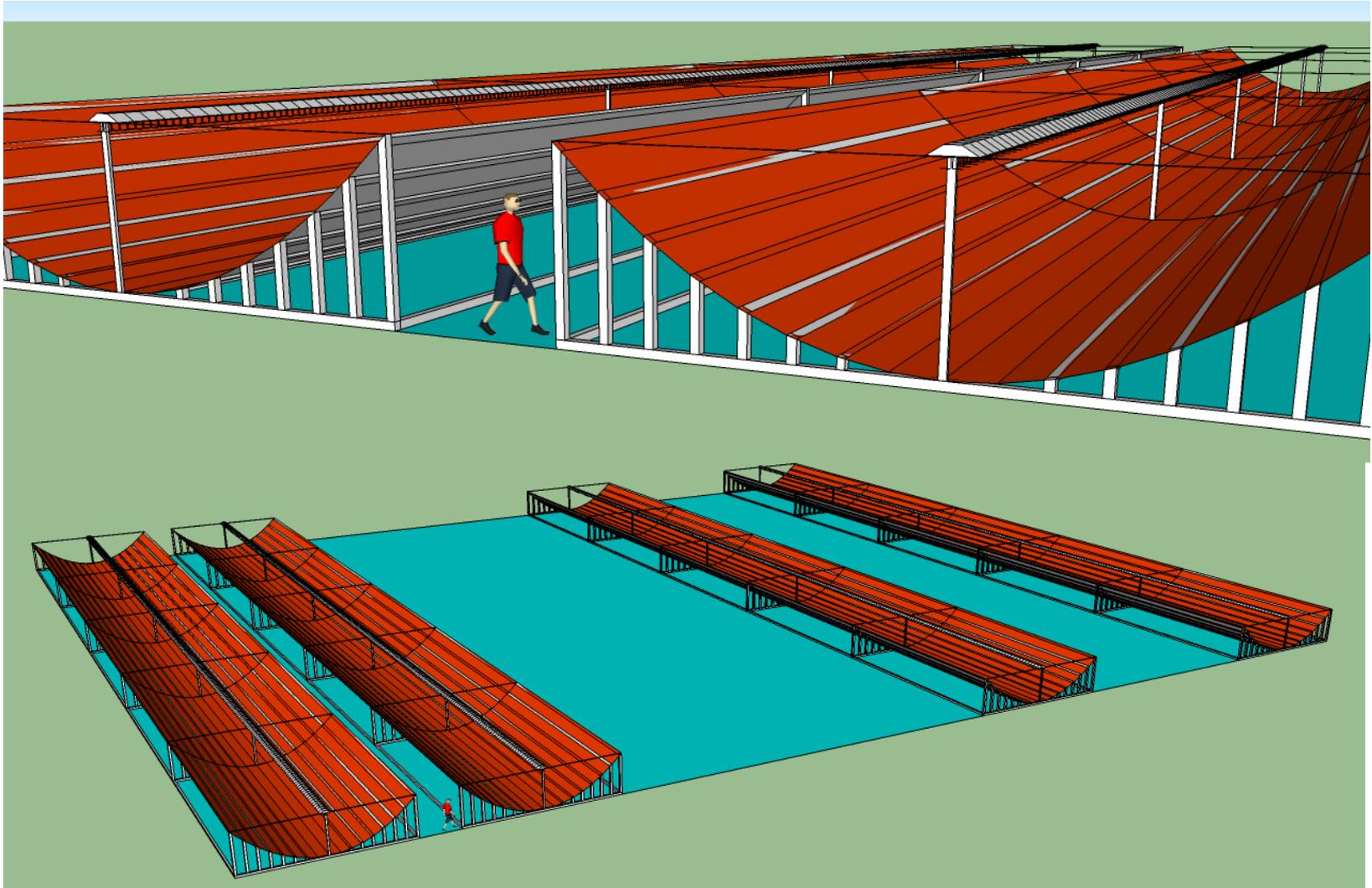
- FFT Radio Telescopes are just recently possible because of:
  - Advances in room temperature, wideband, low noise electronics developed for the cell phone industry
  - High speed transmission (fiber optics, gigabit ethernet, etc.)
  - Availability of low cost, high-speed data processors
    - FFT Processing ( $n \log(n)$ )
    - Field Gate Programmable Arrays (FGPA's)
    - Graphical Processing Units (GPU's)

# The 21cm Cylindrical Radio Telescope (CRT)

- To reduce cost (as a tradeoff of survey speed), the CRT takes the FFT Radio Telescope concept one step further by arranging the CRT as an 2-D collection of 1-D arrays operating in drift-scan mode.<sup>2</sup>
  - The 1-D arrays sit at the focal point of cylindrical reflectors aligned to the meridian
  - The CRT consists of at least 2 cylinders
    - Each cylinder is ranges from 75-150m in length by 10-20m in width
    - Each cylinder has on the order of 256-512 channels per polarization
    - Operating at a frequency range of 500-1000MHz
    - Each cylinder costs on the order of 2-5M\$

<sup>2</sup>The Hubble Sphere Hydrogen Survey, J Peterson , K. Bandura, U. Pen -arXiv:astro-ph/0606104

# CRT Concept



# Pittsburgh Prototype



# The 21cm Cylindrical Radio Telescope (CRT)

- The cylinders are oriented north-south and focus the beam in the east west direction with a beam width of 1.5-3 degrees.
- A feed array of 256-512 uniformly spaced receivers (spacing  $\sim 0.3\text{m}$ ) sits along the focal point of the cylinder.
- A spatial Fourier transform of the  $N$  receiver voltages along a given cylinder produces a fan of  $N$  beams for that cylinder
- The  $k$ th “visibility” is formed by taking the product of the  $k$ th beam from Cylinder A with the  $k$ th beam of Cylinder B
- At each frequency bin, the  $k$ th “visibility” for all  $N$  beams for all possible cylinder pairs is time averaged and recorded.
- The nominal number of cylinders is four.
  - The cylinders are not uniformly spaced in the east-west direction.
  - They are located at positions 1,2,5, & 7 to form 6 effective visibilities for each  $k$ th beam.

# CRT Features

- Low cost
  - Focusing in one direction
  - no moving parts
  - Maintenance & operation advantage (no moving parts)
- Higher stability
  - fixed w.r.t. ground (side-lobes do not change)
  - instrument response averages over right ascension
  - Reflector consistency - gravity is constant
  - Experience at other large radio telescopes show that drift scanning provides the superior stability that is required for large area surveys.

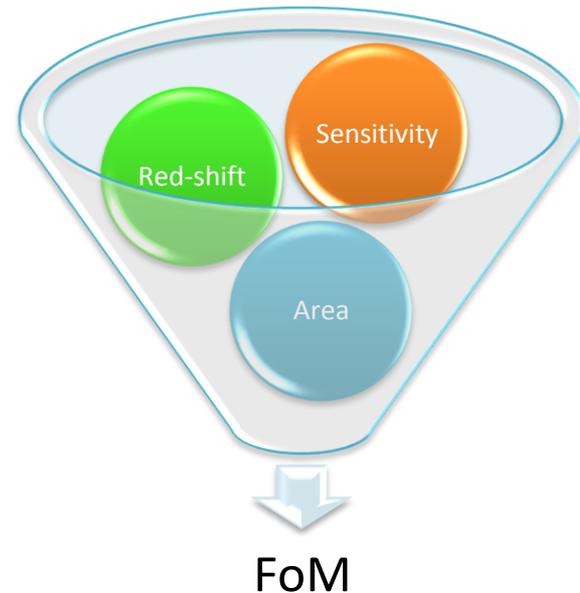
# REQUIREMENTS

# Requirement Optimization

- The purpose of the CRT collaboration is to develop a pre-conceptual design report that describe the “strawman” design
  - For the purpose of the review we will outline a couple of “strawman” design possibilities.
- To focus the collaboration we have developed a web application to evaluate parameter sets
  - Scientific Parameters (SCI)
  - Static Engineering Parameters (STE)
  - Dynamic Engineering Parameters (DYE)
  - Derived Engineering Parameters (DRE)

# Scientific Parameters (SCI) (a.k.a. the 5 magic numbers)

- We want to have a set of numbers that
  - Describe the science
  - Can be derived from **ANY** telescope configuration
- The magic numbers for determining dark energy parameters using BAO
  - Minimum red-shift
  - Maximum red-shift
  - Survey area
  - Pixel Resolution
  - Pixel Sensitivity



# Scientific Parameters (SCI)

- We developed a Monte-Carlo and Fisher matrix technique to analyze the effects of noise and resolution on extracting the BAO signal (Hee-Jong Seo, etal. arXiv:0910.5007, submitted to ApJ)

Number	Description	Symbol
SCI.01	Maximum Red-shift	$Z_{max}$
SCI.02	Minimum Red-shift	$Z_{min}$
SCI.03	Angular Resolution	$\delta\psi$
SCI.04	Survey Area	$A_S$
SCI.05	Sensitivity per Pixel	$\delta T_p$
SCI.06	Figure of Merit with Plank Priors	$FoM_p$
SCI.07	Figure of Merit with Stage II Dark Energy Priors	$FoM_{II}$

# Static Engineering Parameters (STE)

- The static engineering parameters are independent parameters that are
  - important in describing the telescope
  - not easily changed for design optimization
- such as the latitude of the telescope site, amplifier temperature, etc.

# Static Engineering Parameters (STE)

Number	Description	Symbol
STE.01	Survey Time	$\tau_s$
STE.02	Observing Duty Factor	$D_f$
STE.03	Latitude of telescope site	$\alpha_L$
STE.04	Average Sky Temperature	$T_s$
STE.05	Maximum Frequency Span per band	$\Delta F_{b_{max}}$
STE.06	Maximum Fractional Bandwidth per band	$\delta_{f_b}$
STE.07	Number of Polarizations	$N_p$
STE.08	Antenna Feed Power Efficiency	$g_a$
STE.09	Cylinder Width / Cylinder Spacing	$x_{cyl}$
STE.10	Equivalent Amplifier Temperature	$T_A$
STE.11	Electronics Cost per Channel	$R_e$
STE.12	Feed Structure Cost per meter	$R_f$
STE.13	Reflector Cost per Cylinder volume	$R_r$

# Dynamic Engineering Parameters (DYE)

- Dynamic engineering parameters are independent parameters that can be easily varied during the design stage
  - such as feed spacing and the number of channels per cylinder

Number	Description	Symbol
DYE.01	Center Frequency of both bands combined	$F_c$
DYE.02	Average Feed Spacing	$D_f$
DYE.03	Number of digital channels per cylinder per polarization	$N_f$
DYE.04	Average Number of possible cylinder locations	$N_L$
DYE.05	Average Cylinder packing factor	$p_f$
DYE.06	Target Cost	$C_T$

# Derived Engineering Parameters (DRE)

- Derived engineering parameters are design specific parameters
  - such as cylinder length and width
  - but are derived from the static and dynamic engineering parameters.

# Derived Engineering Parameters (DRE)

Number	Description	Symbol
DRE.01	Number of Cylinders	$N_c$
DRE.02	Cylinder Length	$L_c$
DRE.03	Cylinder Width	$W_c$
DRE.04	Cylinder Spacing	$S_c$
DRE.05	Declination Span	$\Delta\theta_d$
DRE.06	Feed Length	$h_f$
DRE.07	Feed Spacing	$d_f$
DRE.08	Band Center Frequency	$F_{cb}$
DRE.09	Wavelength	$\lambda$
DRE.10	Band Frequency Span	$\Delta F_b$
DRE.11	Resolution Bandwidth	$\delta f$
DRE.12	Minimum Digital Memory	$M_d$
DRE.14	Integration Time per Pixel	$\tau_p$
DRE.15	Number of Channels per polarization	$N_{fT}$
DRE.16	Electronics Cost	$C_e$
DRE.17	Feed Structure Cost	$C_f$
DRE.18	Reflector Cost	$C_R$
DRE.19	Total Cost	$C_T$

# Telescope Cost

- It is not intended that these costs include everything that would arise in designing and building a large radio telescope
  - such as site preparation, non-recoverable engineering costs, overhead, contingency etc.,
- These costs should only be used in trying to compare sets of design parameters.
- The cost of the digital electronics is assumed to scale only with the number of feeds:

$$C_e = N_f N_c N_p R_e$$

# Telescope Cost

- The cost of the telescope structure is broken into two parts.
- The feed line is the most complicated part of the reflector system and this cost will scale as the total length of the array.

$$C_f = L_c N_c R_f = N_f N_c d_f R_f$$

- The cost of the main reflector surface will not only be proportional to area
  - but height as well since tall structures will be more difficult to build.
  - For a fixed f-ratio, the height will scale with cylinder width.

$$C_r = L_c N_c W_C^2 R_f = N_f N_c d_f W_C^2 R_f$$

# Requirement Web Application

CRT Design Requirements II

Calculate FoM      Optimize      Iterations

	Band 1			Target	Step	Band 2			
SCI.01 - Redshift Range	1.8	1.33	1			1	0.67	0.43	
SCI.02 Survey Area	3.64	2.81	2.41			3.64	3.05	2.58	pi Steradians
SCI.04 Angular Resolution	17.11	14.26	12.22			18.33	15.28	13.09	arc-min
SCI.05 Sensitivity per Pixel	87.37	104.74	194.42			74.76	91.53	172.33	uK
SCI.06 Plank Priors Figure of Merit		89.67		89.67			89.67		
SCI.07 DE II Priors Figure of Merit		235.84		235.84			235.84		
DYE.01 Center Frequency	600			<input type="text" value="740"/>	<input type="text" value="0"/>	840			MHz
DYE.02 Feed Spacing	0.5838			<input type="text" value="0.6"/>	<input type="text" value="0"/>	0.5449			lambda
DYE.03 Digital Channels per Cylinder per Polarization	413			<input type="text" value="413"/>		413			
DYE.04 Number of Cylinder locations	6			<input type="text" value="5"/>	<input type="text" value="2"/>	4			
DYE.05 Cylinder Packing Factor	66.67			<input type="text" value="60"/>	<input type="text" value="20"/>	100			%
DYE.06 Total Cost	15.01			<input type="text" value="15.0"/>		13.31			M\$
STE.01 Survey Time	2			<input type="text" value="2.0"/>		2			years
STE.02 Observing Duty Factor	50			<input type="text" value="50.0"/>		50			%
STE.03 Latitude	35			<input type="text" value="35.0"/>		35			degrees

Microsoft PowerPoint - [FermilabScienceReq.pptx]

# Requirement Web Application

CRT Design Requirements II

http://crt21cm3.fnal.gov:8080/CrtMagicNumbers/magicNumberII.jsp

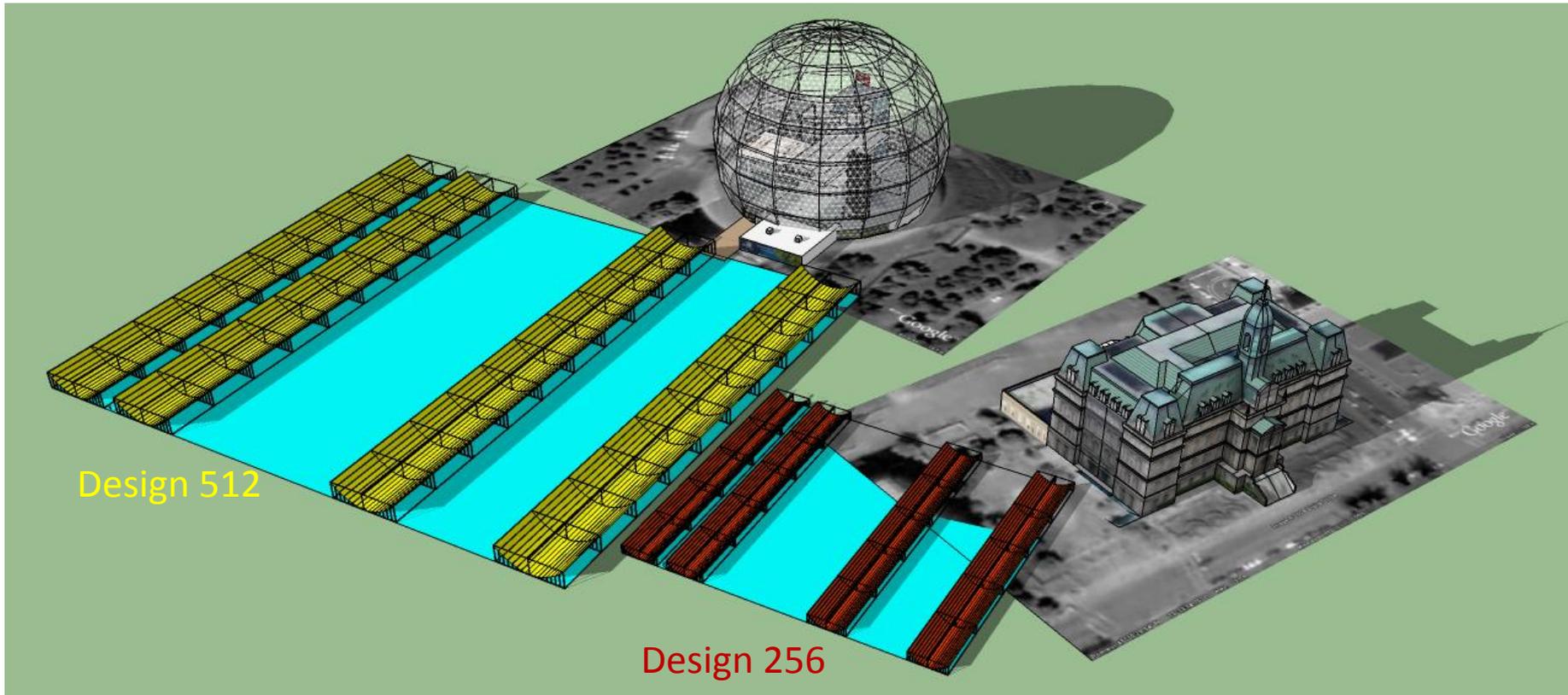
STE.08	Antenna Efficiency	80		80.0		80		%	
STE.09	Antenna Width Fill Factor	80		80.0		80		%	
STE.10	Amplifier Temperature	50		50.0		50		K	
STE.11	Electronics Cost per Channel	3000		3000.0		3000		\$	
STE.12	Feed Structure Cost Rate	2300		2300.0		2300		\$/meter	
STE.13	Reflector Volume Cost Rate	32		32.0		32		\$/meter^3	
DRE.01	Number of Cylinders	4				4			
DRE.02	Cylinder Length	120.55				80.37		meters	
DRE.03	Cylinder Width	16.07				16.07		meters	
DRE.04	Cylinder Spacing	20.09				20.09		meters	
DRE.05	Declination Span	180	117.85	94.47		180	133.17	103.73	degrees
DRE.06	Feed Length	29.19				19.46		cm	
DRE.07	Feed Spacing	29.19				19.46		cm	
DRE.08	Frequency	500	600	700		700	840	980	MHz
DRE.09	Wavelength	60	50	42.86		42.86	35.71	30.61	cm
DRE.10	Frequency Span		200				280		MHz
DRE.11	Res. Bandwidth	2.65	2.07	1.63		2.44	1.73	1.18	MHz
DRE.12	Minimum Digital Memory		245				472		
DRE.13	Integration Time per Pixel	8.02	7.16	2.64		5.74	5.39	2.23	days
DRE.14	Number of Channels per polarization		1652				1652		
DRE.15	Electronics Cost		9.91				9.91		M\$
DRE.16	Feed Structure Cost		1.11				0.74		M\$
DRE.17	Reflector Volume Cost		3.99				2.66		M\$

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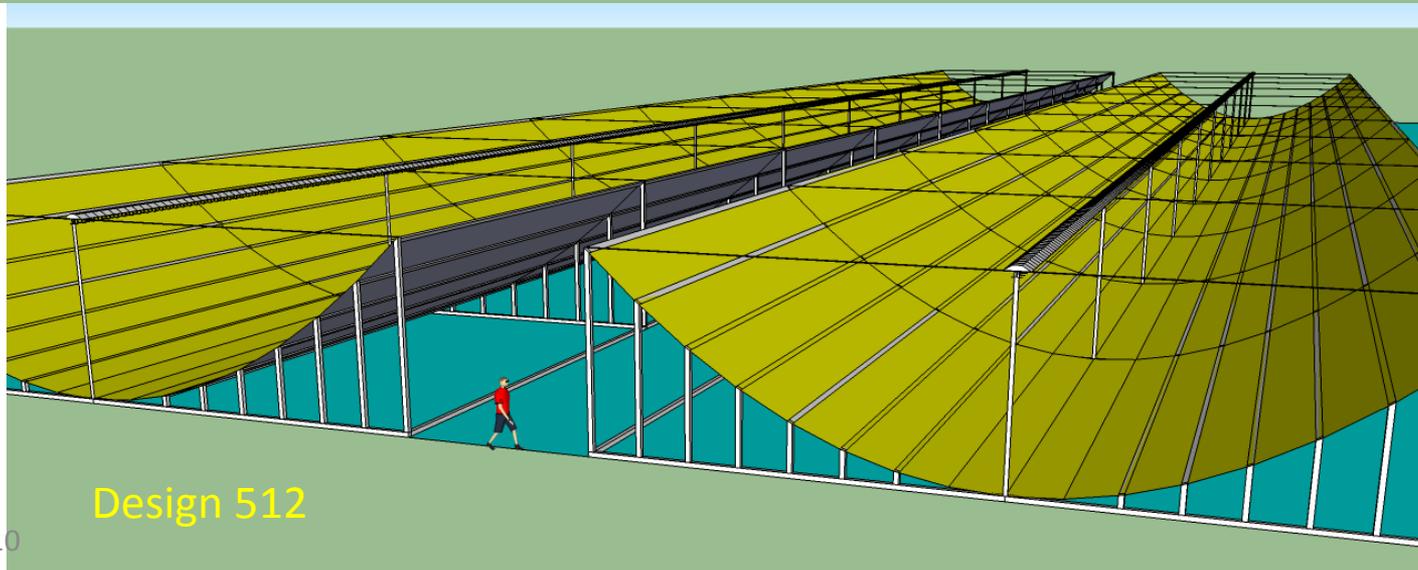
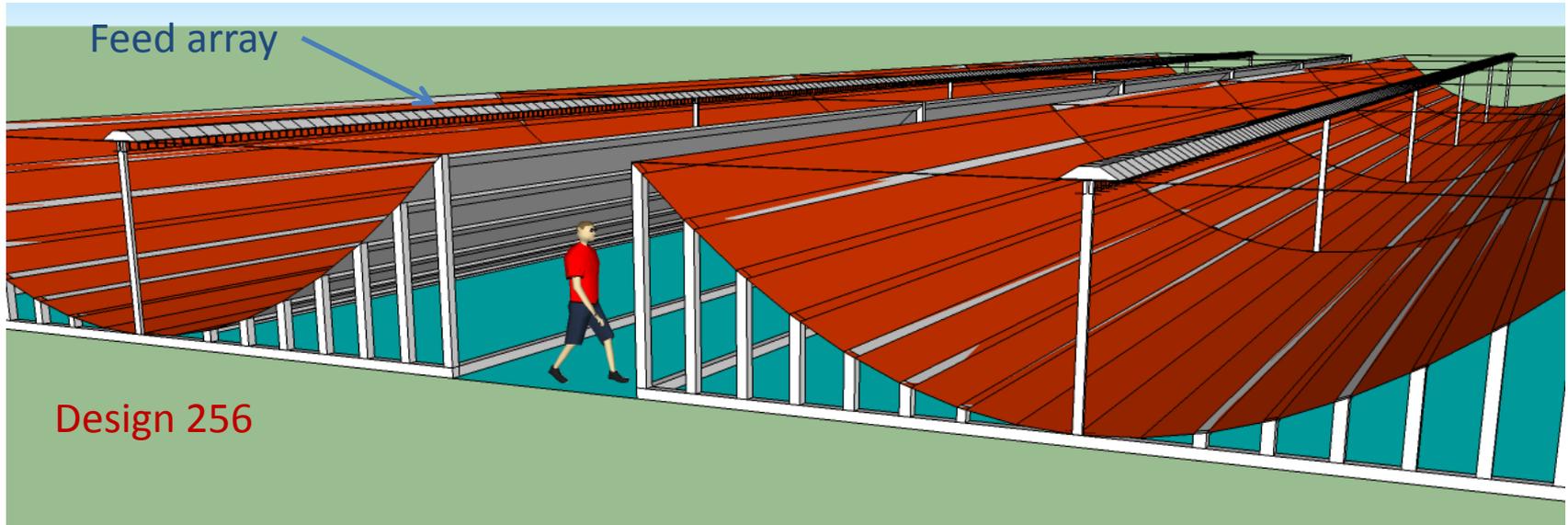


# STRAWMAN DESIGNS

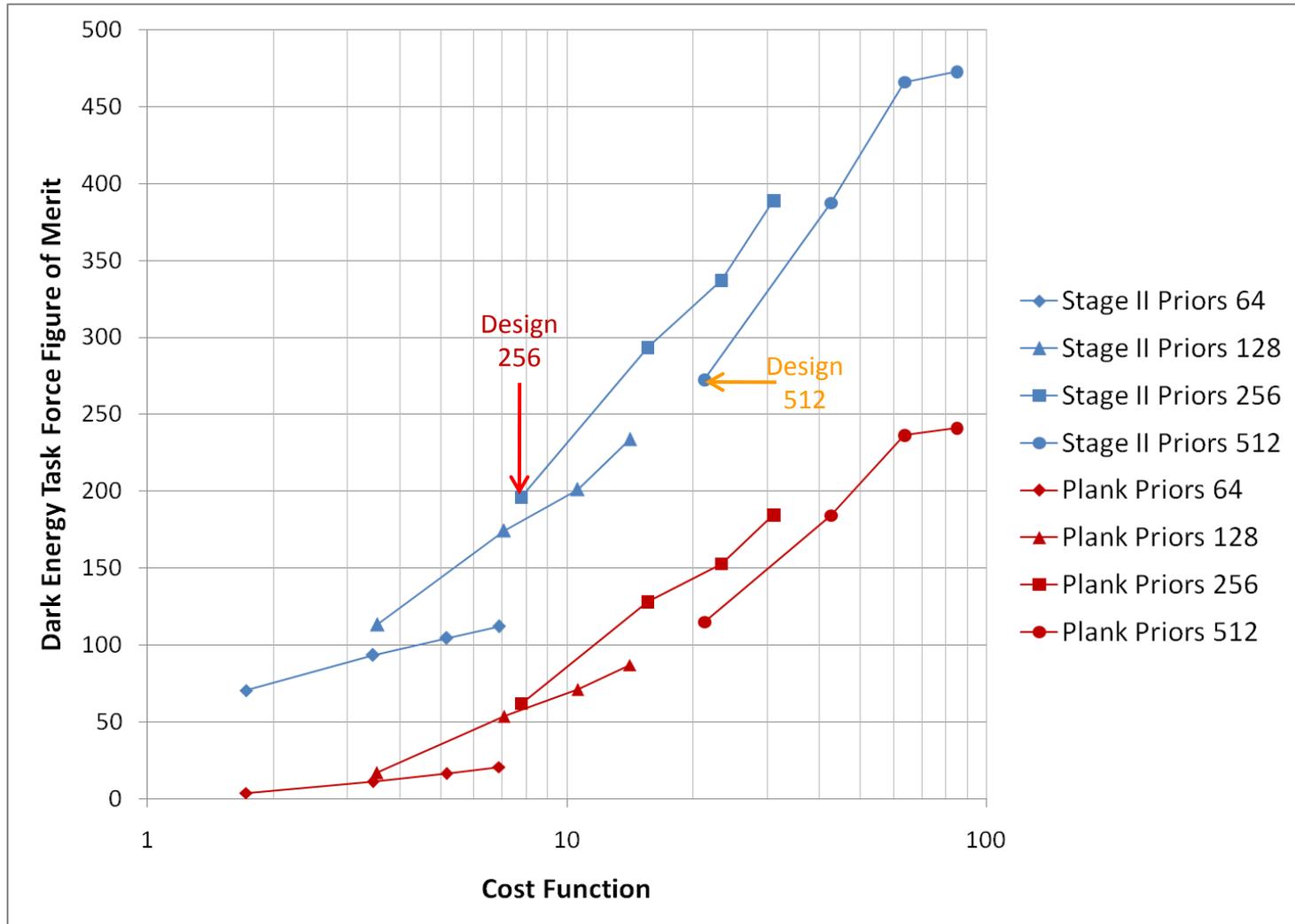
# CRT Strawman Designs



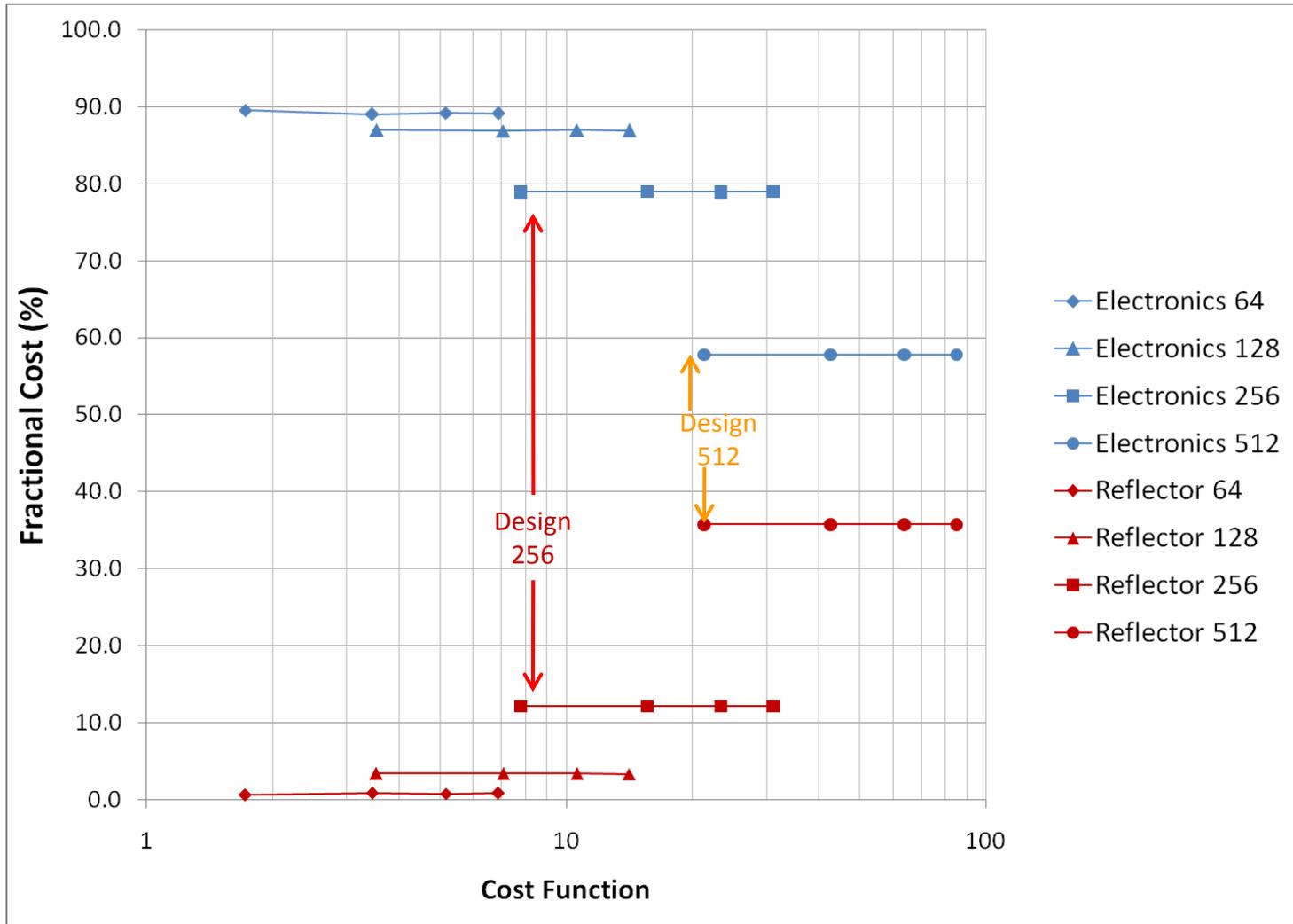
# CRT Strawman Designs



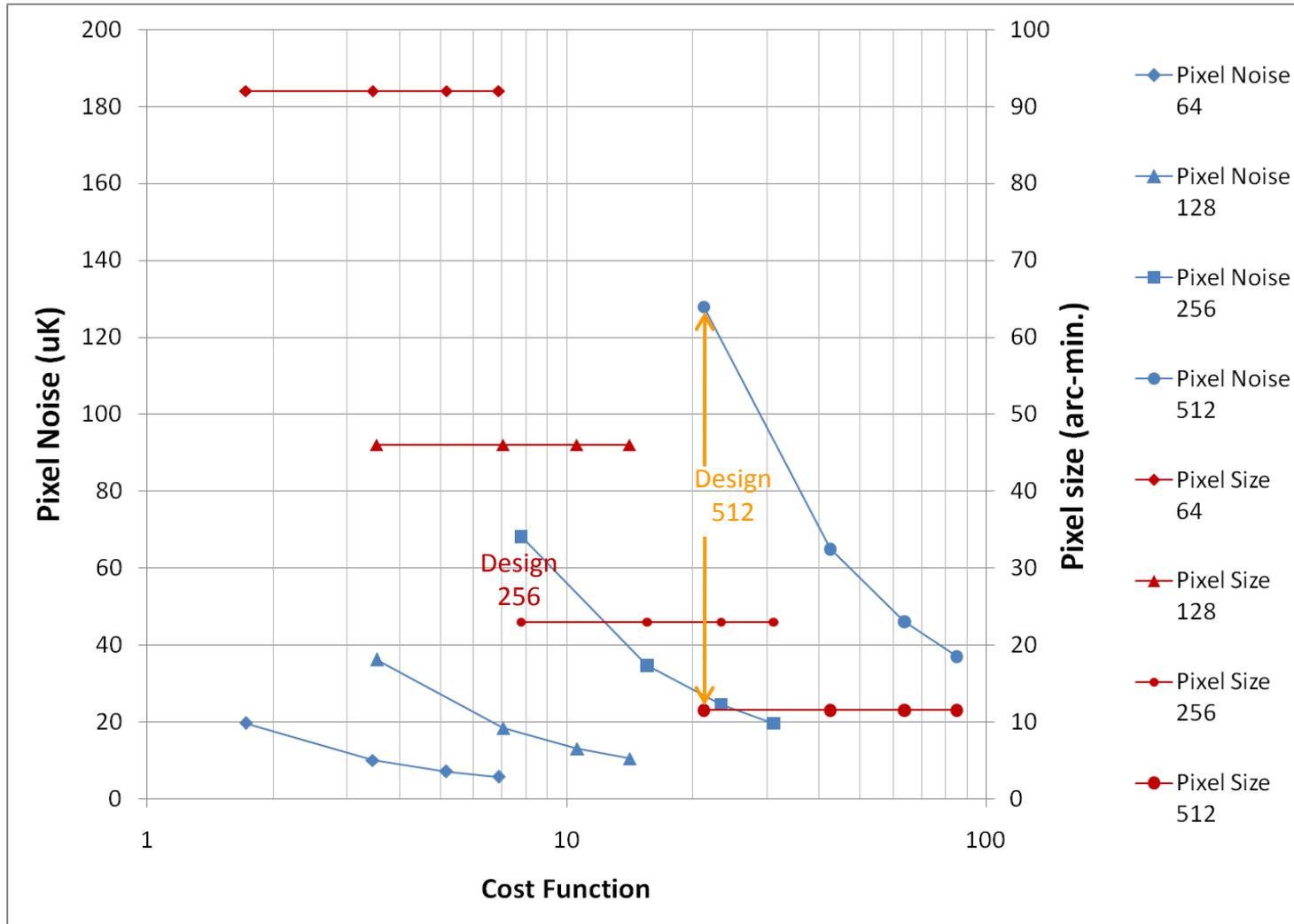
# Figure of Merit vs Cost



# Fractional Cost



# Pixel Noise and Resolution



# Requirement Summary

- A Dark Energy Task Force Figure of Merit of
  - 200 can be obtained with a CRT that “costs” ~8M\$
  - 270 can be obtained with a CRT that “costs” ~21M\$



# **INSTRUMENT SIMULATION**

# Foreground Removal

- The straw-man designs presented previously were analyzed assuming foreground removal will be successful
- Foregrounds consist of galactic synchrotron emission, point radio sources, etc.
- To tackle foreground removal, we are developing a large package of software to simulate the telescope and test foreground removal.

# Goals of the Telescope Simulations

- How well do we need to know the telescope (transfer function)?
- What is the optimal foreground removal algorithm?
- How pathological can the sky be and still recover the BAO signal?

# Sky Simulation Software

- Simulate telescope response
  - Include the effects of noise and telescope errors
  - Provide data stream that would mimic the real telescope
- Reconstruct the sky temperature from telescope response
- Foreground removal algorithms

# Code Suite

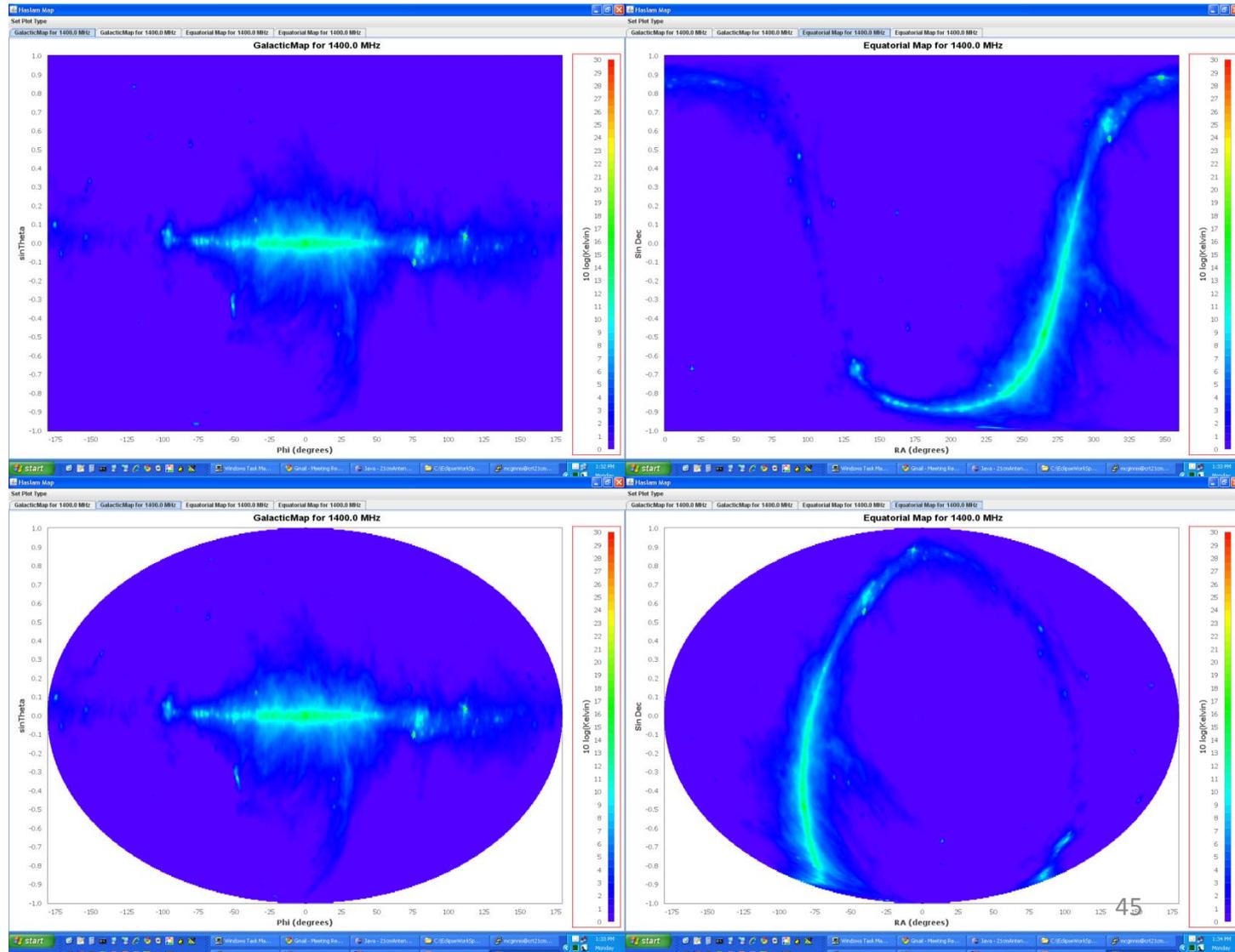
- 29 Java classes organized into 4 packages
- Major Packages
  - Sky Map Generator
  - Cylinder Visibility Simulator
    - Include the effects of noise and telescope errors
    - Provide data stream that would mimic the real telescope
  - Telescope Transfer function
    - Cylinder Visibility Modeler
    - Sky Reconstructor
  - Foreground Removal Algorithms

# **SKY MAP GENERATOR**

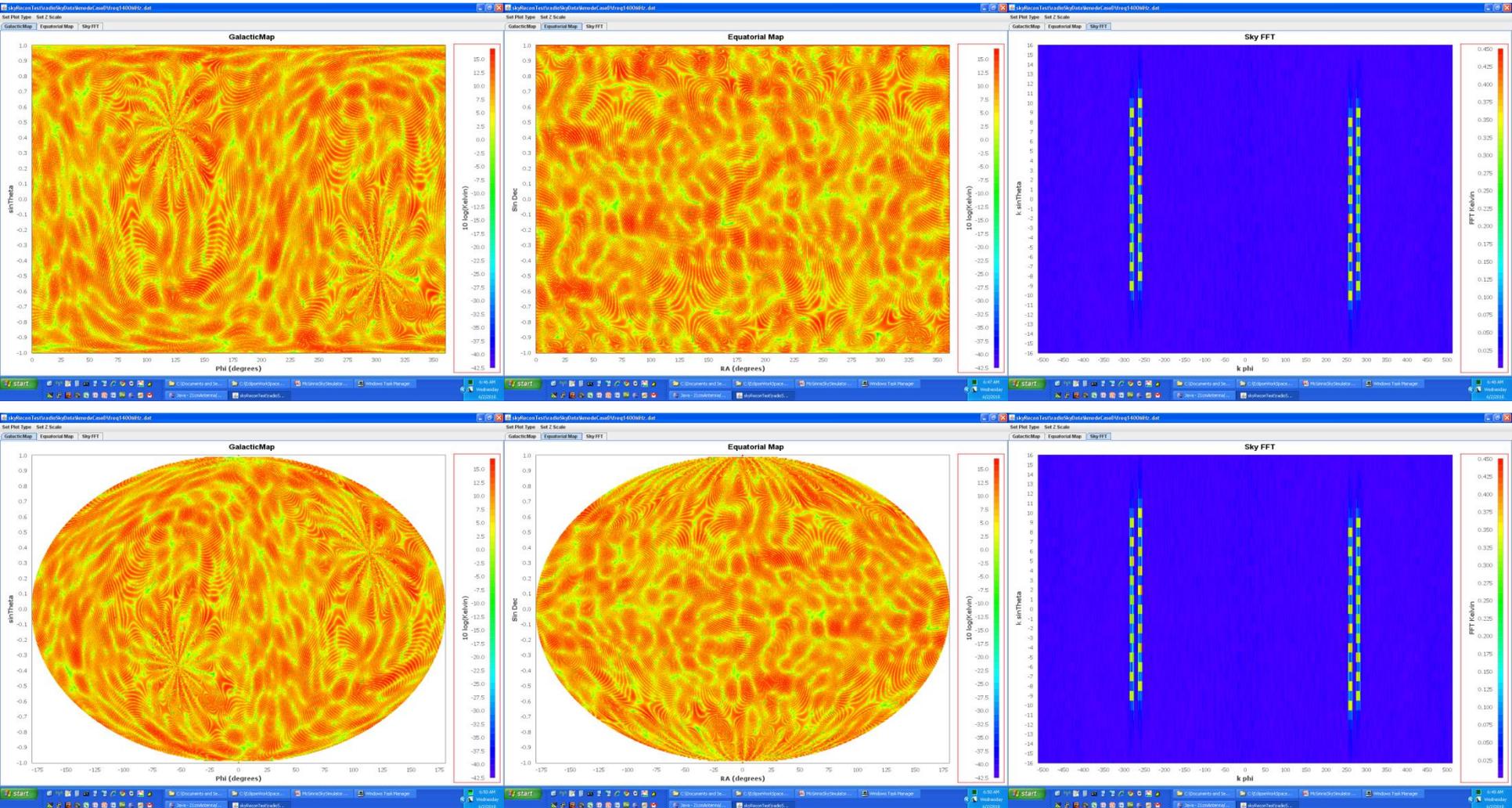
# Sky Map Generator Plotter for Haslam

## Sky Map at 1.4 GHz

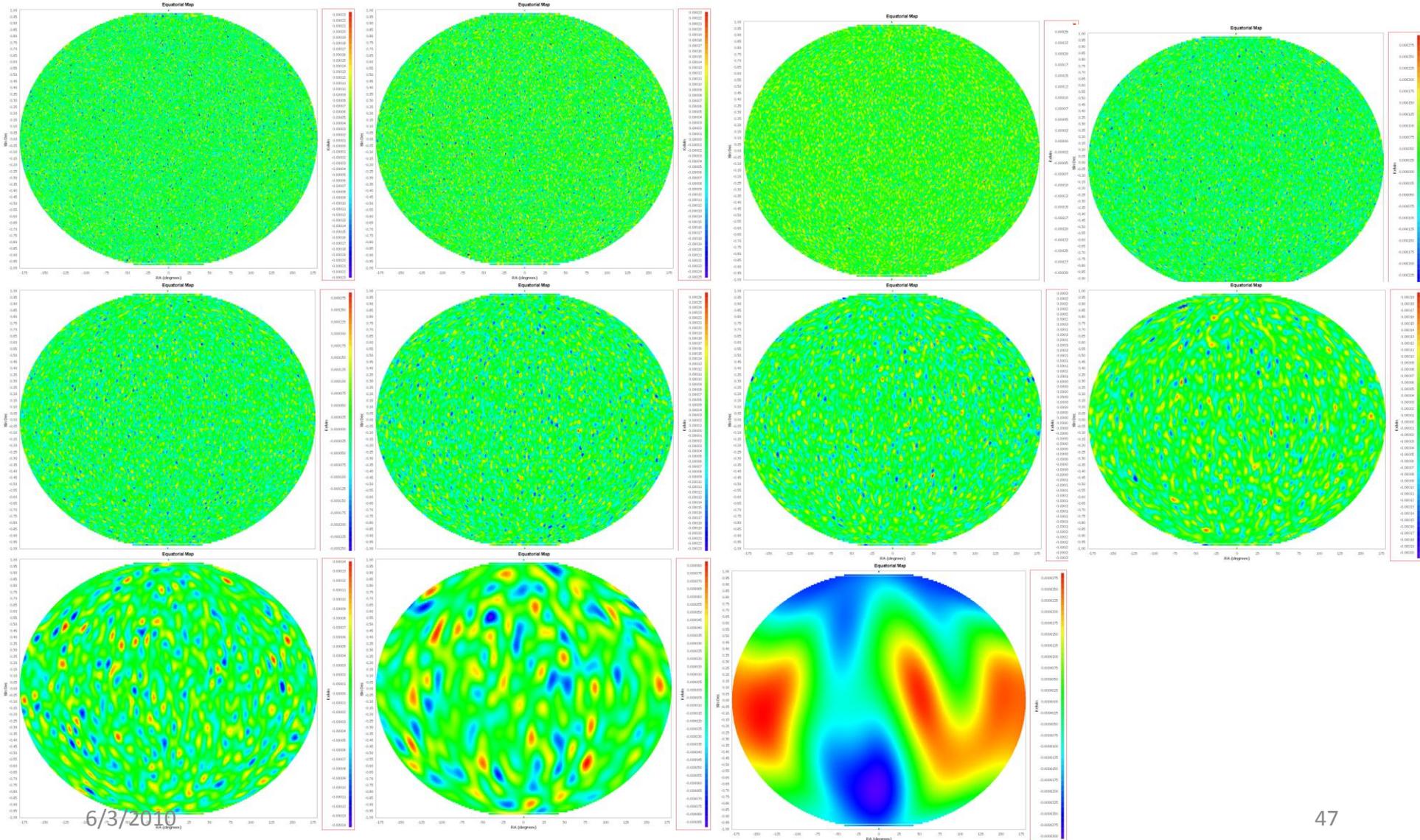
- Maps in Healpix format
  - $N_{\text{side}} = 512$
- Maps use MIT Angelica 10 parameter frequency fit



# Designer Skies



# BAO Signal First Peak from 400-1400MHz



# CYLINDER VISIBILITY SIMULATOR

# Cylinder Visibility Formulation

## Formulation of Cylinder Visibilities

Dave McGinnis  
November 5, 2009

### Feed Amplitude

The voltage received at a feed located at coordinate  $\mathbf{r}$  is:

$$\frac{v(\vec{r})}{\sqrt{2R}} = \iint_{\Omega} f(\Omega) a(\Omega) e^{-j\vec{\beta}(\Omega) \cdot \vec{r}} d\Omega \quad (1)$$

The sky flux amplitude is:

$$|f(\Omega) d\Omega|^2 = \frac{kT_{sky}(\Omega)}{\lambda^2} d\Omega_{pow} \quad (2)$$

where  $d\Omega_{pow}$  is the differential power solid angle area. The incoming wave vector is:

$$\vec{\beta}(\Omega(\theta, \phi)) = \frac{2\pi}{\lambda} (\sin(\theta)\hat{x} + \cos(\theta)\sin(\phi)\hat{y}) \quad (3)$$

The collecting area of the feed is:

$$A(\Omega) = |a(\Omega)|^2 \quad (4)$$

The noise power generated by the feed amplifier is:

$$P_z = |p_z|^2 \quad (5)$$

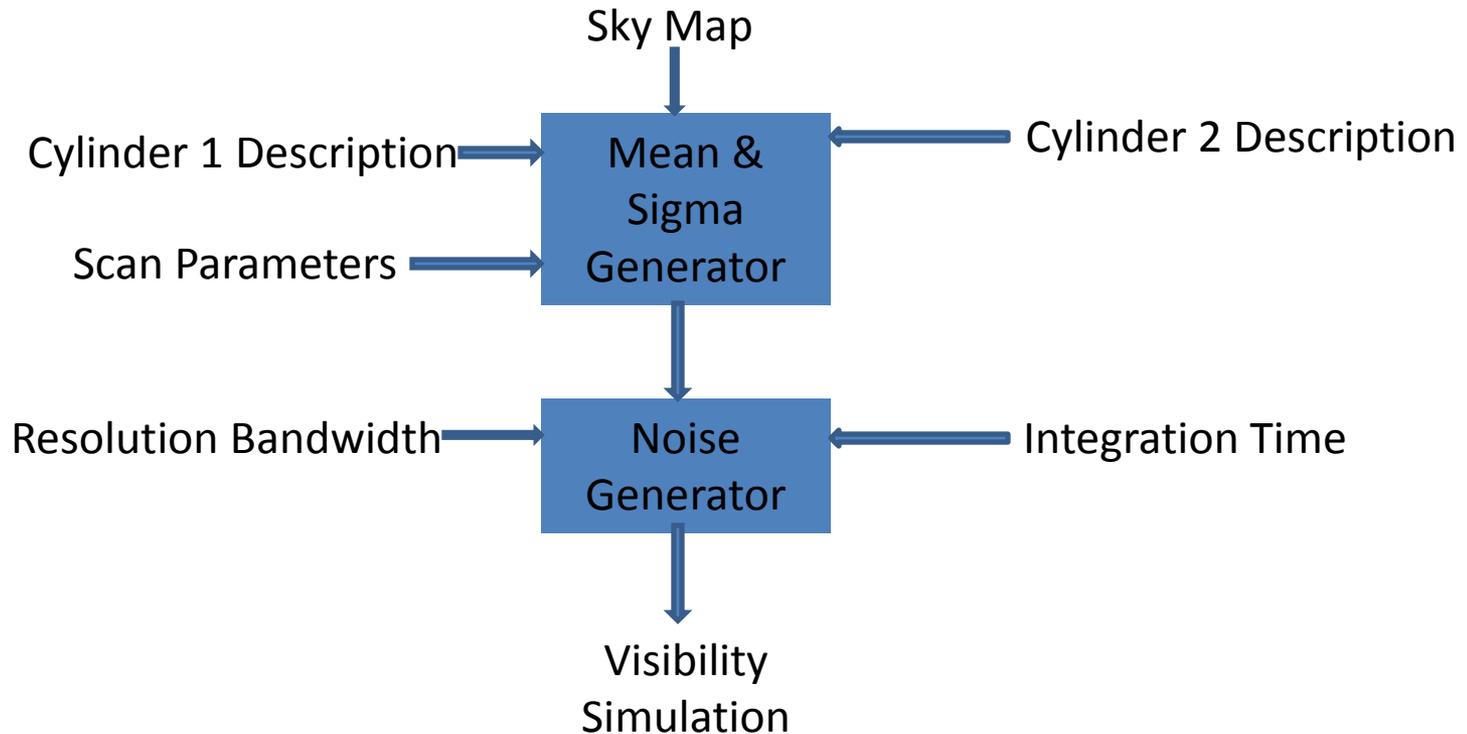
If the sky is pixelized into  $\mathbf{q}$  pixels then, the signal amplitude at feed  $\mathbf{n}$  of cylinder  $\mathbf{m}$  is:

$$p_{n,m} = p_{z_{n,m}} + \sum_q \Delta\Omega_q f_q a_{q,n,m} e^{-j\vec{\beta}_q \cdot \vec{r}_{n,m}} \quad (6)$$

### Cylinder Amplitude

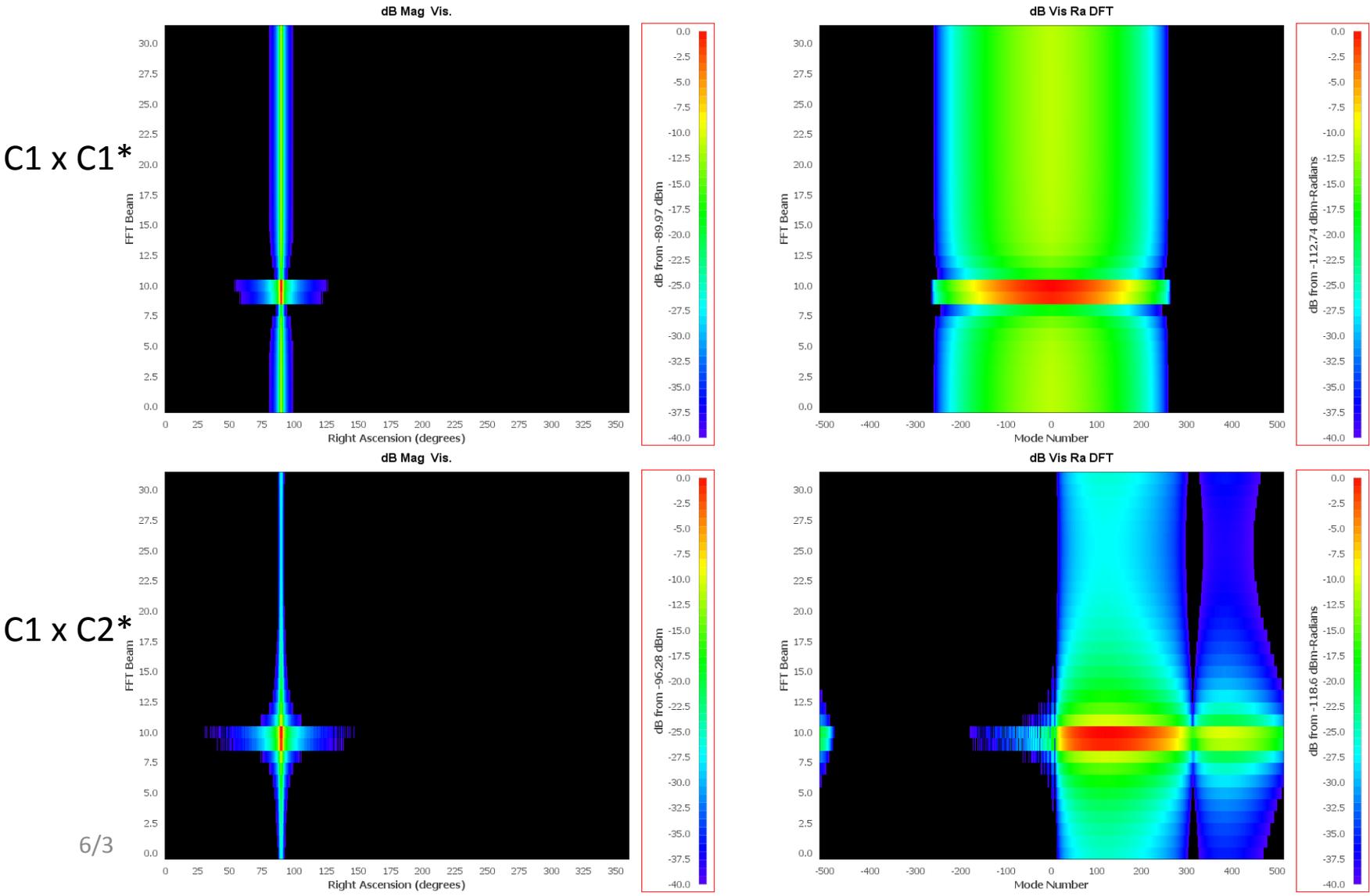
A spatial Fourier transform will be taken of the cylinder feed voltage.

# Cylinder Visibility Simulator





# Noiseless Pittsburgh Cylinder Visibility of a Point Source



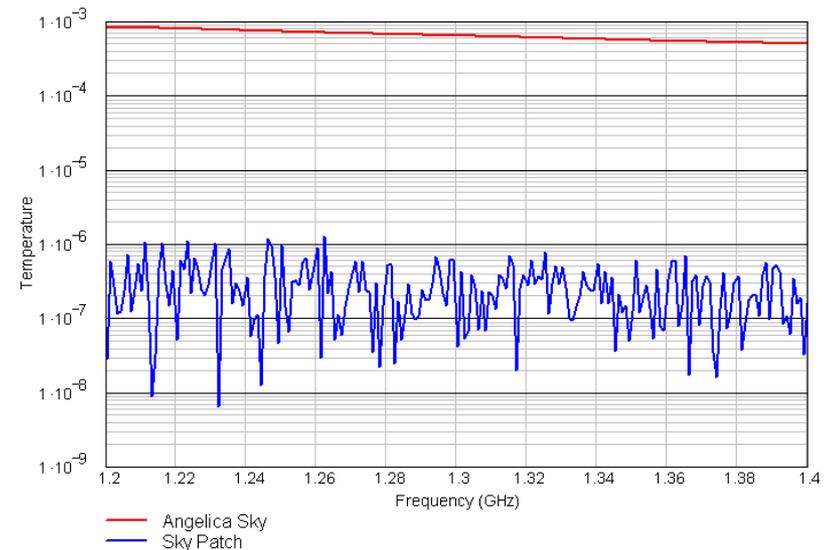
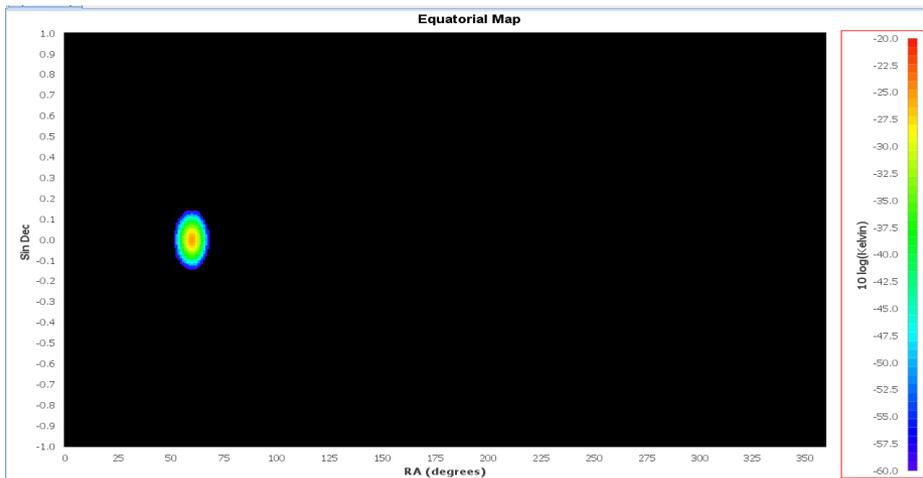
# **FORE-GROUND SUBTRACTION USING CYLINDER VISIBILITIES**

# Frequency-Smoothed Sky Subtraction Algorithm

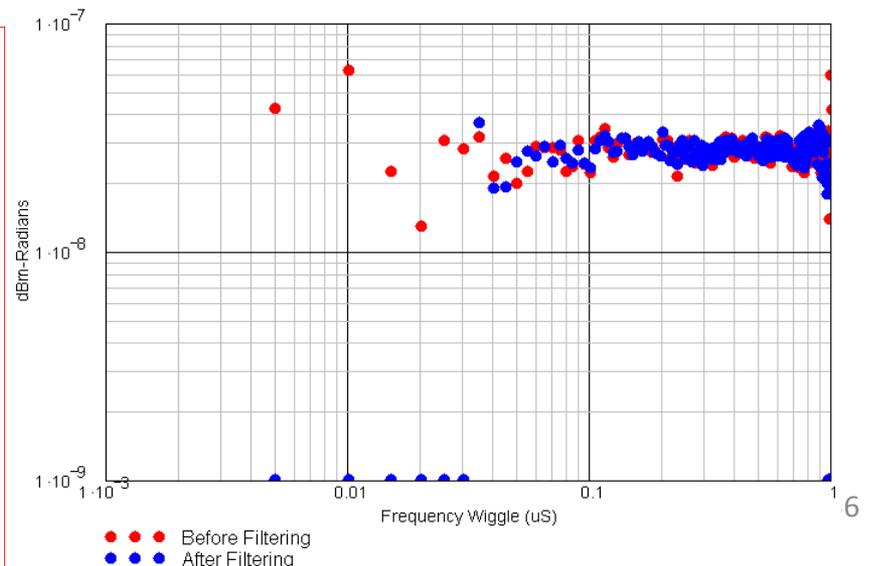
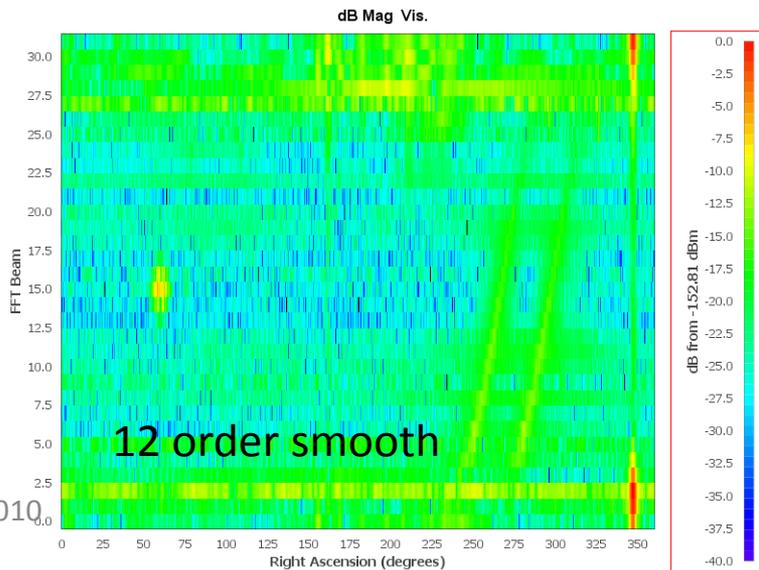
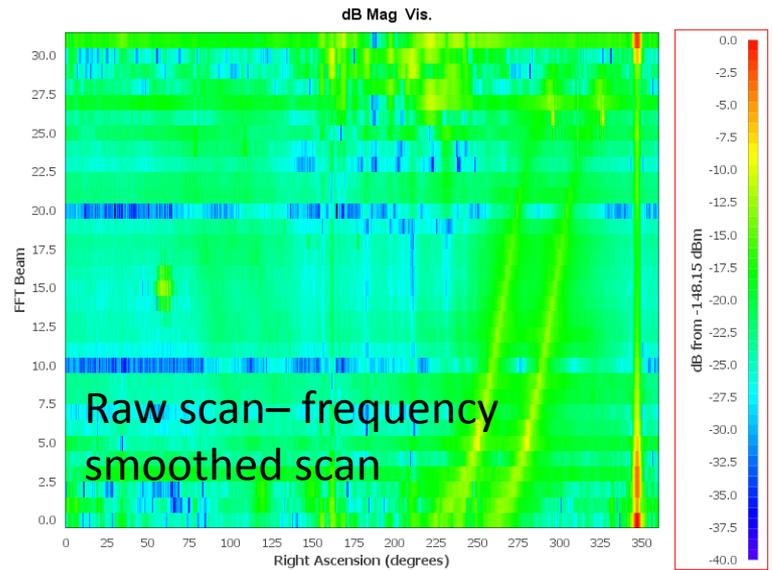
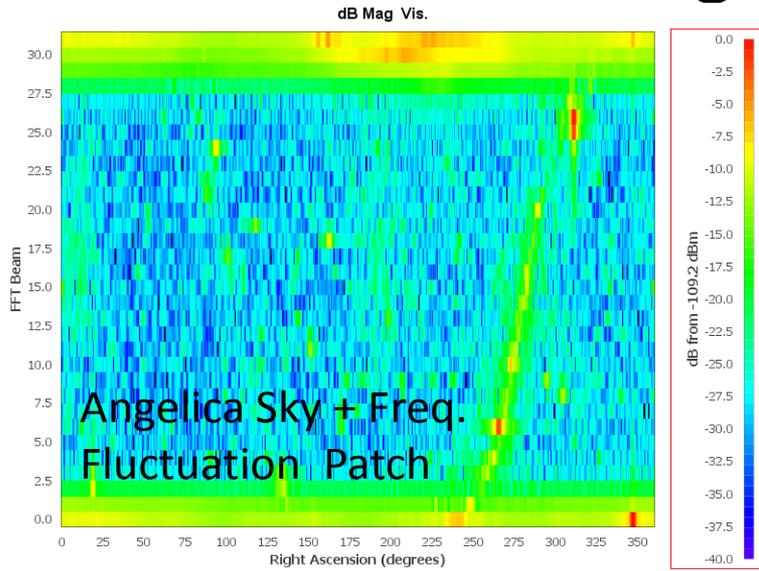
- This is just an example to demonstrate problems of Mode Mixing
  - Not trying to address the issues of sky frequency smoothness pathology
- The “cleanest” algorithm would be to attempt to remove the foreground directly from the raw visibility data
- Algorithm
  - Take cylinder visibility data smooth it along the frequency axis using a N order polynomial for each pixel
  - Subtract the frequency-smoothed map from the raw map producing a difference map
  - From the difference map, fit each visibility spectrum “pixel” as a nth order polynomial in frequency
  - Subtract the frequency-smoothed pixel trace from the difference map pixel by pixel
  - Further FFT filter in frequency each the remaining pixel trace

# Frequency Fluctuating Sky Patch Test

- As a simple test, a “fluctuating” sky patch was added to frequency smooth angelica sky.
- The patch had a white “noise” spectrum of  $\sim 1\text{mK}$  and an rms beam width of 3 degrees



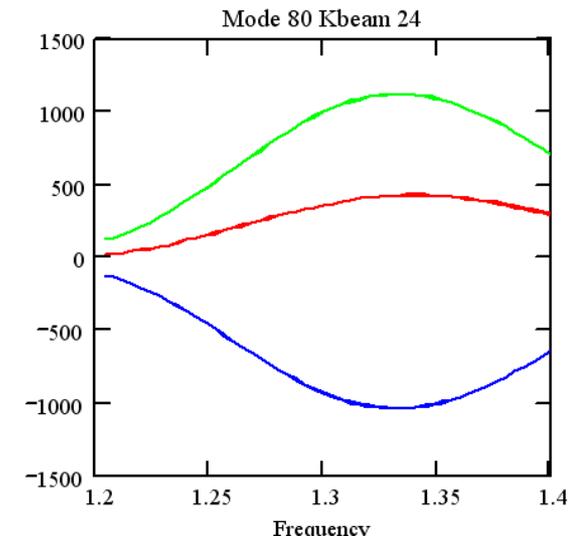
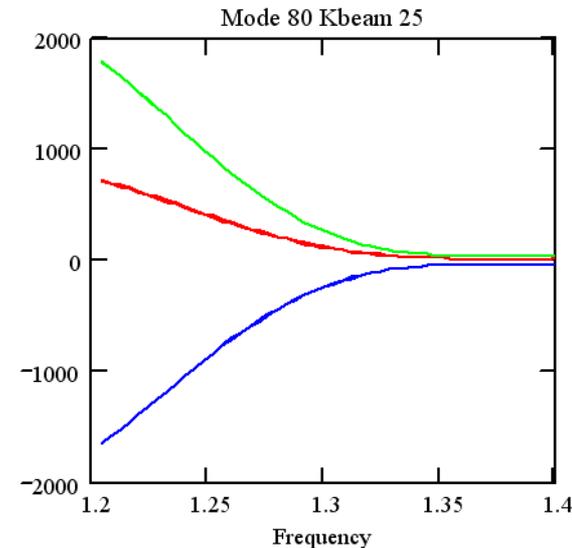
# Fluctuating Sky Path Simulations of the Pittsburgh Cylinders



# Mode Mixing Smoothness

- The location of the FFT beams on the sky changes with frequency
  - Objects seem to “move” from beam to beam as a function of frequency – termed “mode mixing”
  - The algorithm works because the beams moves smoothly
  - However, lots of fitting “horsepower” is used up in tracking the beams
- If we have an accurate model of the telescope:
  - We know how the beams will move on the sky
  - We can reconstruct the sky and remove the mode mixing.

“Hot Pixel” tracks  
vs Frequency



# **CYLINDER VISIBILITY MODELER AND K-SPACE SKY RECONSTRUCTION**

# Cylinder Modeler

## Sky Reconstruction from Cylinder Visibilities

Dave McGinnis  
June 1, 2010

### Visibility

This note will consider the reconstruction of the sky from the measured visibilities from a pair of cylinder antenna arrays. It is assumed that the cylinders fixed and are oriented along the meridian. Each cylinder is populated with N feeds spaced uniformly along the length. The output voltage of each feed provides an input of a spatial Fourier transform along the cylinder length. The spatial Fourier transform forms N beams along the length of the cylinder.

For a pair of cylinders the visibility between cylinders is formed for each beam. As the sky drifts through the cylinder beam, the visibility for beam k is:

$$v_k(\varphi) = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta, \phi)}{\lambda^2} T(\theta, \varphi - \phi) \cos(\theta) d\theta d\phi \quad (1)$$

Where  $\varphi$  is the time of the day (in units of angle),  $\lambda$  is the wavelength and  $T$  is the power flux of the sky. The cylinder pair Fourier area is defined as

$$\tilde{A}_k(\theta, \phi) = \tilde{a}_{k,c1}(\theta, \phi) (\tilde{a}_{k,c2}(\theta, \phi))^* \quad (2)$$

where the subscripts c1, c2 indicate cylinder 1 and cylinder 2, respectively. The Fourier root area of a cylinder is defined as

$$\tilde{a}_{k,c}(\theta, \phi) = \sum_n a_n(\theta, \phi) e^{-j\beta(\theta, \phi) \cdot r_{n,c}} e^{j2\pi k \frac{r_n}{N}} \quad (3)$$

Where  $n$  is the feed number,  $r_{n,c}$  is the global location of the feed and  $\beta$  is the incoming wave vector:

$$\hat{\beta}(\theta, \phi) = \frac{2\pi}{\lambda} (\sin(\theta)\hat{x} + \cos(\theta)\sin(\phi)\hat{y}) \quad (4)$$

It is assumed that the length of the cylinders is in the x direction.

### Sky Expansion

Since the sky is periodic, it can be expanded in a Fourier series:

$$T_c(\theta, \phi) = \sum_l \sum_m \hat{T}_{m,l} e^{jl\pi \sin(\theta)} e^{jm\phi} \quad (5)$$

Since  $T_c$  is a complex function but the sky temperature must be a real function, the sky temperature can be written as:

$$T(\theta, \phi) = T_c(\theta, \phi) + (T_c(\theta, \phi))^* \quad (6)$$

$$T(\theta, \phi) = \sum_l \sum_m \hat{T}_{m,l} e^{jl\pi \sin(\theta)} e^{jm\phi} + \sum_l \sum_m (\hat{T}_{m,l})^* e^{-jl\pi \sin(\theta)} e^{-jm\phi} \quad (7)$$

$$T(\theta, \phi) = \sum_l \sum_m (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*) e^{jl\pi \sin(\theta)} e^{jm\phi} \quad (8)$$

Substituting Equation 8 into Equation 1,

$$v_k(\varphi) = \sum_l \sum_m (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*) e^{jm\varphi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta, \phi)}{\lambda^2} e^{jl\pi \sin(\theta)} e^{-jm\phi} \cos(\theta) d\theta d\phi \quad (9)$$

### Fourier Transform of Cylinder Visibilities

Now assume that the visibility is measured at N discrete times during the day. The visibility will be periodic with a period of a day so we can expand the visibility into a Fourier series.

$$v_k(\varphi_n) = \sum_{m'} \tilde{V}_{k,m'} e^{jm'\varphi_n} \quad (10)$$

where

$$\tilde{V}_{k,m'} = \frac{1}{N} \sum_n v_k(\varphi_n) e^{-jm'\varphi_n} \quad (11)$$

Substituting Equations 9 into Equations 11,

$$\tilde{V}_{k,m} = \sum_l \tilde{A}_{k,l,m} (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*) \quad (12)$$

where:

$$\tilde{A}_{k,l,m} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta, \phi)}{\lambda^2} e^{jl\pi \sin(\theta)} e^{-jm\phi} \cos(\theta) d\theta d\phi \quad (13)$$

The discrete approximation for Equation 13 is:

$$\tilde{A}_{k,l,m} \approx \frac{4\pi}{N_p N_q} \sum_{q=0}^{N_q} \sum_{p=0}^{N_p} \frac{\tilde{A}_k(\theta_p, \phi_q)}{\lambda^2} e^{jl(\pi \sin(\theta))_p} e^{-jm\phi_q} \quad (14)$$

The matrix form of Equation 13 is:

$$[\tilde{V}_m] = [\tilde{A}_m][\tilde{T}_m] \quad (15)$$

### Multiple Visibilities

There can be a number of combination of cylinder visibilities that have a non-zero component a given right ascension mode m. However, the sky mode temperature at mode m must be unique.

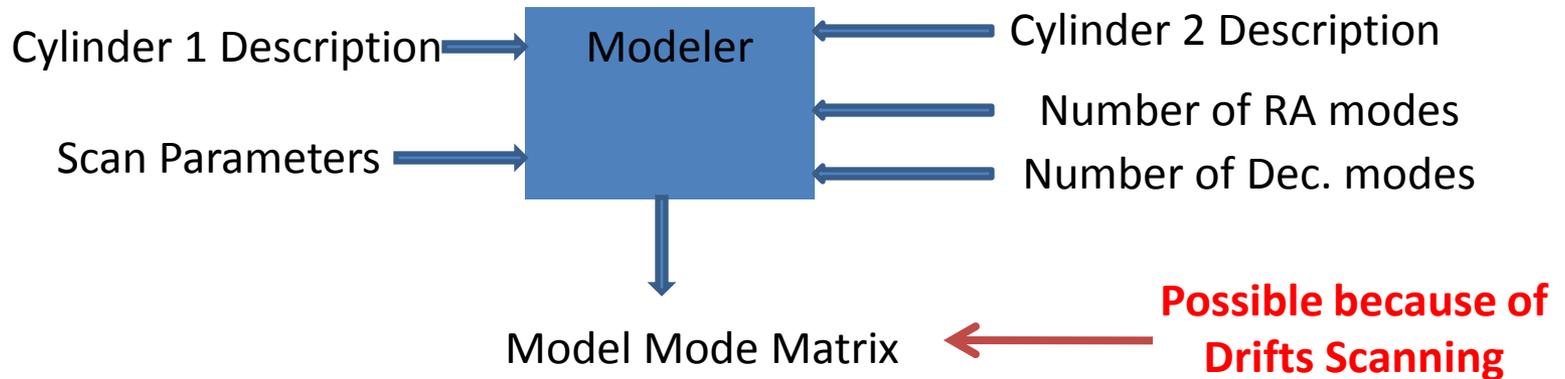
$$[\tilde{V}_m]^{(1)} = [\tilde{A}_m]^{(1)} [\tilde{T}_m] \quad (16)$$

$$[\tilde{V}_m]^{(2)} = [\tilde{A}_m]^{(2)} [\tilde{T}_m] \quad (17)$$

To satisfy both equations in a least squares fit, equations 16 and 17 can be combined:

$$\begin{aligned} & \left\{ \left\{ [\tilde{A}_m]^{(1)} \right\}^T \right\}^* [\tilde{V}_m]^{(1)} + \left\{ \left\{ [\tilde{A}_m]^{(2)} \right\}^T \right\}^* [\tilde{V}_m]^{(2)} \\ & = \left( \left\{ \left\{ [\tilde{A}_m]^{(1)} \right\}^T \right\}^* + \left\{ \left\{ [\tilde{A}_m]^{(2)} \right\}^T \right\}^* \right) [\tilde{T}_m] \end{aligned} \quad (18)$$

# Cylinder Visibility Modeler



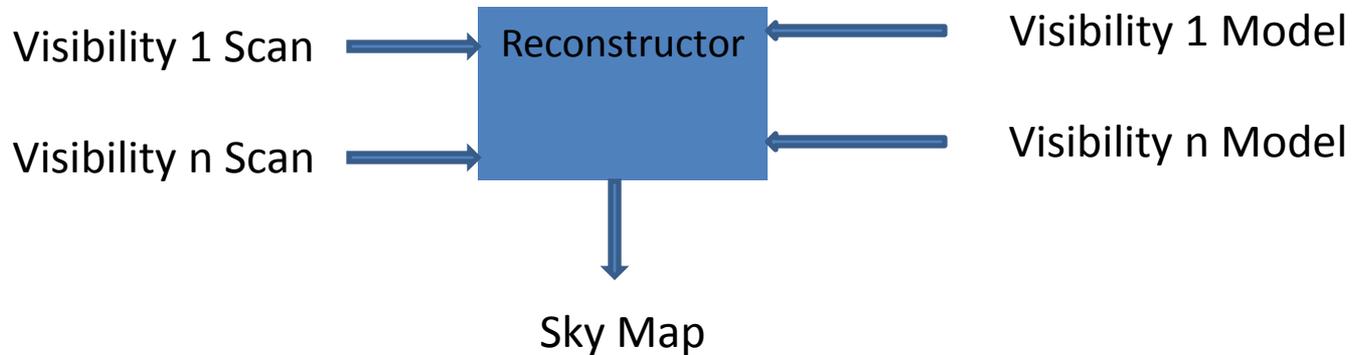
$$\tilde{V}_{k,m'} = \frac{1}{N} \sum_n v_k(\varphi_n) e^{-jm\varphi_n}$$

$$T(\theta, \phi) = \sum_l \sum_m (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*) e^{jl\pi \sin(\theta)} e^{jm\phi}$$

$$\hat{A}_{k,l,m} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\tilde{A}_k(\theta, \phi)}{\lambda^2} e^{jl\pi \sin(\theta)} e^{-jm\phi} \cos(\theta) d\theta d\phi$$

$$\tilde{V}_{k,m} = \sum_l \hat{A}_{k,l,m} (\hat{T}_{m,l} + (\hat{T}_{-m,-l})^*)$$

# Sky Reconstruction



$$[\tilde{V}_m]^{(1)} = [\hat{A}_m]^{(1)} [\tilde{T}_m]$$

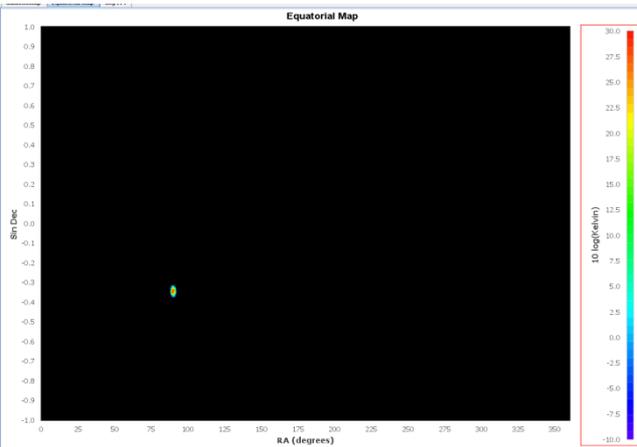
$$[\tilde{V}_m]^{(2)} = [\hat{A}_m]^{(2)} [\tilde{T}_m]$$

$$\begin{aligned} & \left\{ \left\{ [\hat{A}_m]^{(1)} \right\}^T \right\}^* [\tilde{V}_m]^{(1)} + \left\{ \left\{ [\hat{A}_m]^{(2)} \right\}^T \right\}^* [\tilde{V}_m]^{(2)} \\ &= \left( \left( \left\{ \left\{ [\hat{A}_m]^{(1)} \right\}^T \right\}^* + \left\{ \left\{ [\hat{A}_m]^{(2)} \right\}^T \right\}^* \right) [\tilde{T}_m] \end{aligned}$$

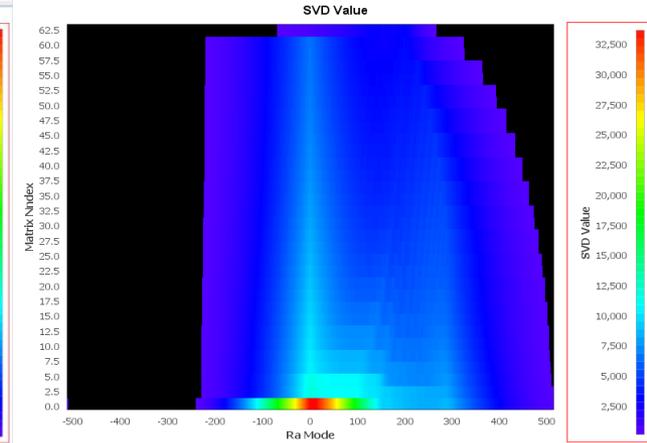
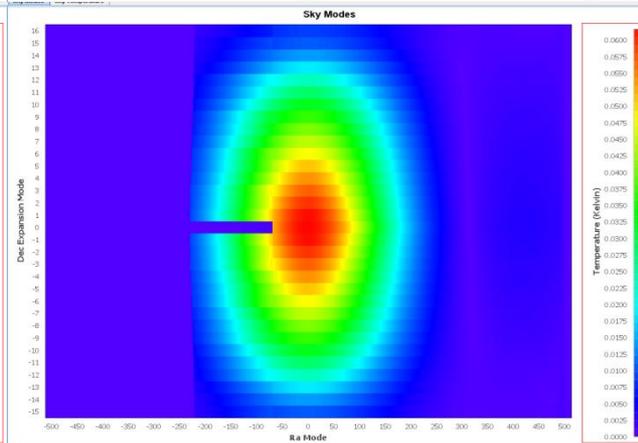
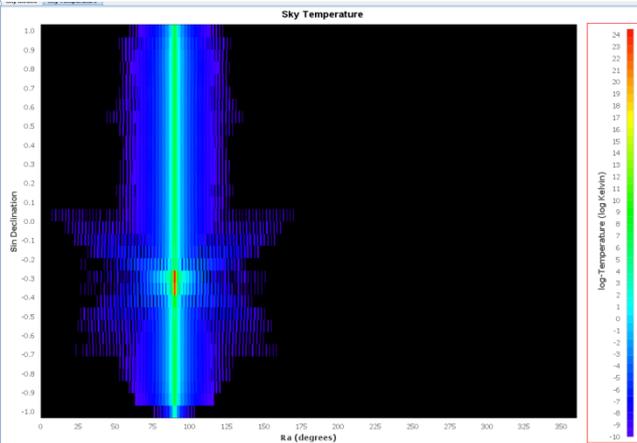
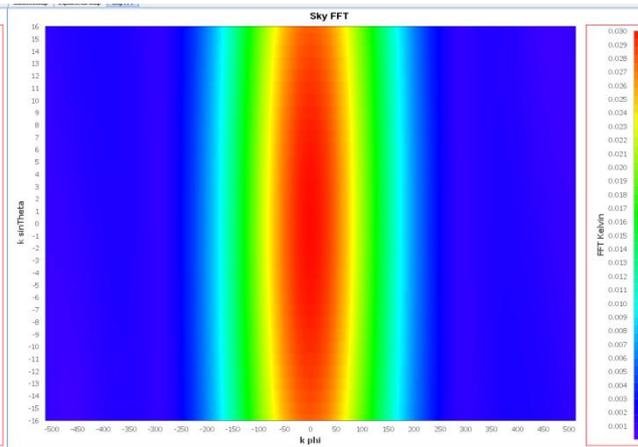
# Point Source Sky Map Reconstruction

(Pittsburgh Cylinders)

Input Sky



Input K-Space



Output Sky  
6/3/2010

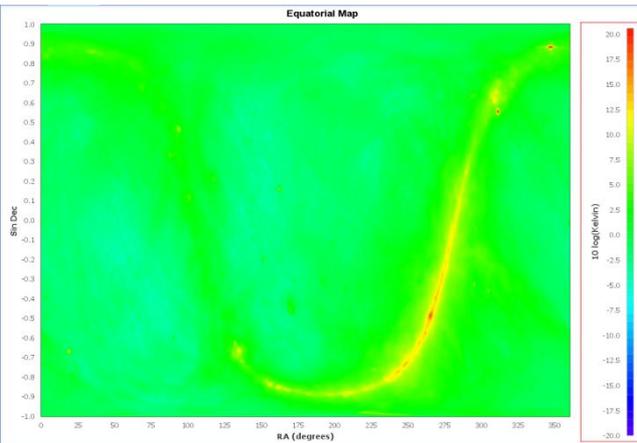
Output K-Space

SVD Values

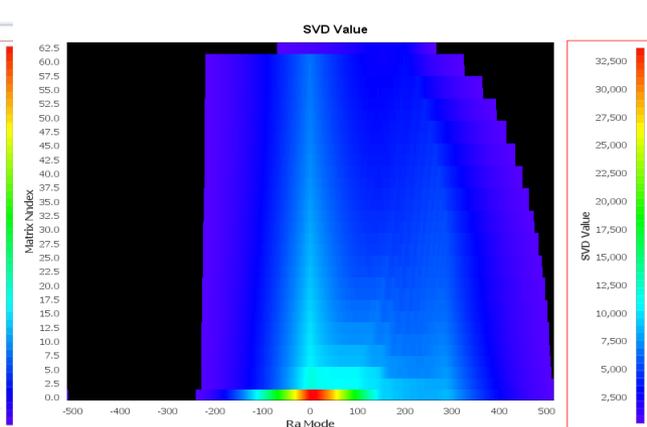
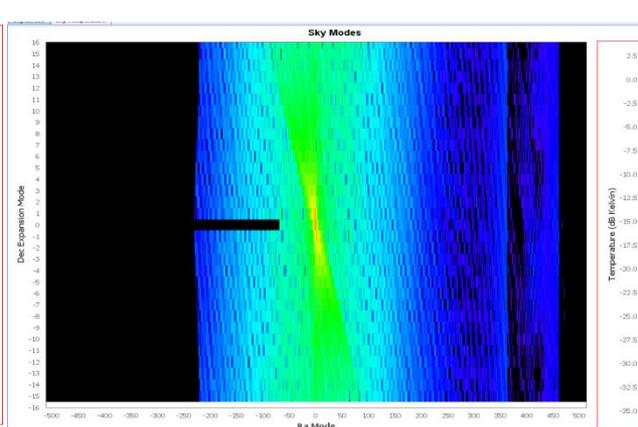
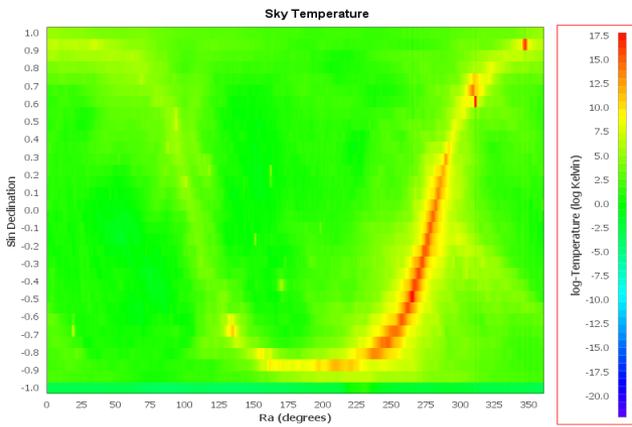
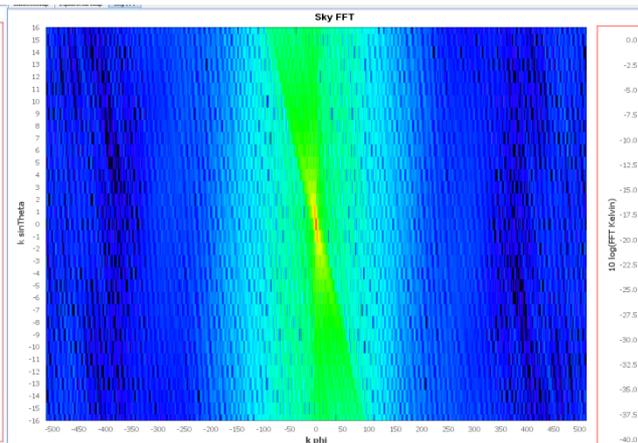
# Angelica Sky Map Reconstruction

(Pittsburgh Cylinders)

Input Sky



Input K-Space



Output Sky  
6/3/2010

Output K-Space

SVD Values

# Foreground Removal using BAO Simulations

- We need  $\sim 128$  feeds/cylinder to resolve the first BAO peak
- The simulation size increases  $\sim$  cubically with the number of feeds
  - 256 feeds and 4 cylinders requires  $\sim 500$  times more computing than 32 feeds and 2 cylinders
  - In addition, the simulation has to be done over a number of frequency slices ( $>100$  slices)
    - To simulate a telescope with 256 feeds/cylinder and 4 cylinders (7 cylinder combinations) at 100 frequency slices requires 1.8 years of cpu time.
    - To reconstruct the telescope, , the telescope model matrices requires a total of 1.500 Terabytes of disk space.

# FNAL-KICP Joint Cluster



Fermi National Accelerator Laboratory

- We have implemented the simulation software on the FNAL-KICP cluster which has >1200 cpu's available.
  - Using 112 cpu's, it takes about 140 hours to simulate the 256 feed telescope with 4 cylinders (7 cylinder combinations).

## FNAL - KICP Joint Cluster

### Hardware Details

Component	Properties	Total
Master node	8-way, 16GB, fulla	2
Quad-socket dual-core nodes	8-way, 128GB, cc001	1
Quad-socket dual-core nodes	8-way, 32GB, cc002-004	3
Dual-socket quad-core nodes	8-way, 16GB, cc005-...	149
Total # of cores		1240
Infiniband nodes		24

```
Usage Totals: 838/1224 Procs, 123/153 Nodes, 120/180 Jobs Running
Node States: 2 down,offline 50 free 92 job-exclusive 9 offline

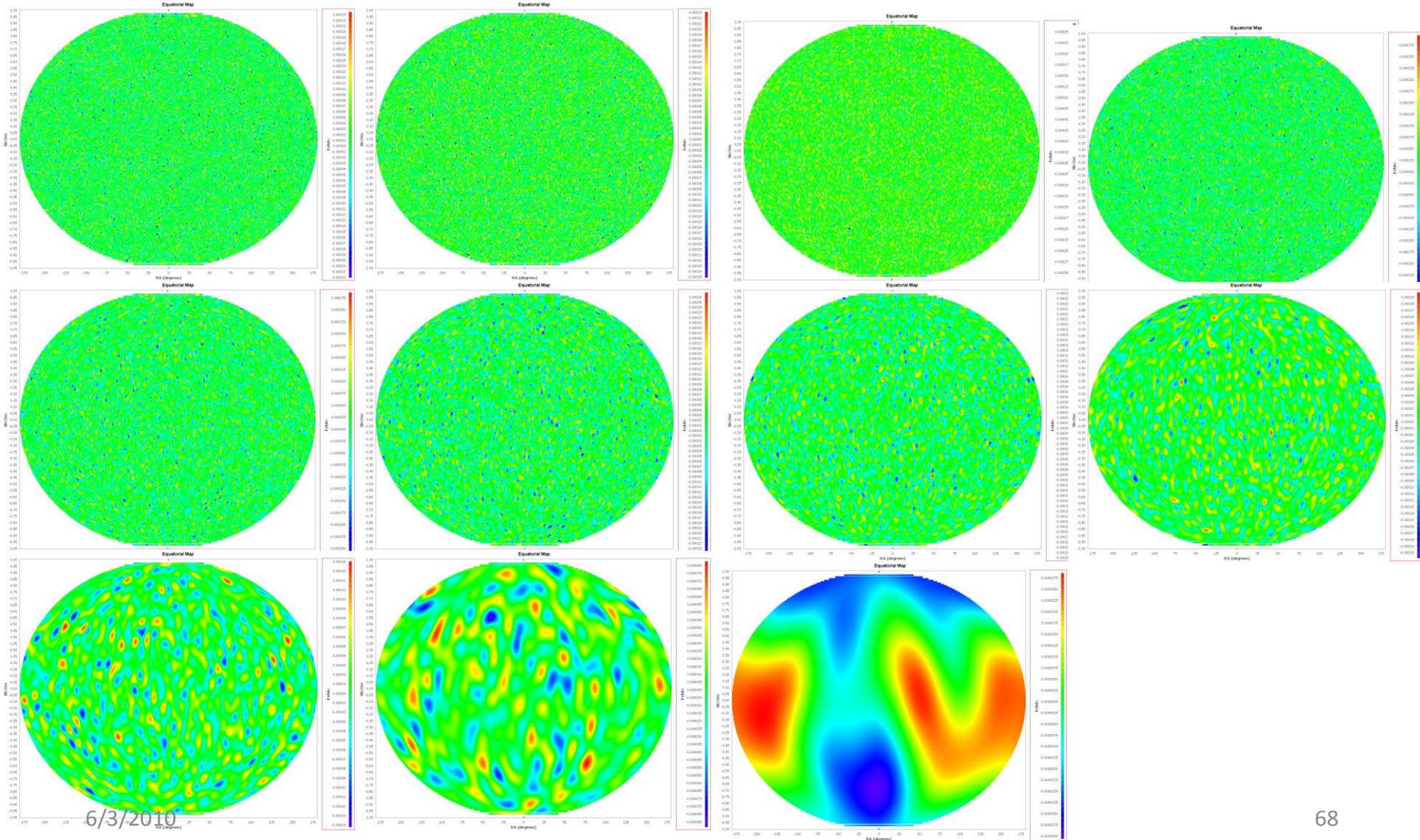
visible CPUs: 0,1,2,3,4,5,6,7
1 2 3 4 5 6 7 8 9 0
-----
cc001 00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000
cc011 vvvvvvvv 00000000 11111111 mmmmmmmi cccccccc mmmmmmmn ci..... k..... dT..... x.....
cc021 p..... AAAAAAAA z..... fggggggg Tggggggg R..... gggggggg ....L... V..... CCCCCC.
cc031 YYYYYYYY N..... 00000000 ssssssss CCCCCC. e..... rrrrrrrr A.....b... G.....
cc041 p..... AAAAAAAA C1CCCC. yyyyyyyy vvvvvvvv V..... yHyyyy... hhhhhhhh HHHHHHHH h.....
cc051 .fffff. dddddd HHHHHHHH ..... ssssssss j..... Pppppppp Pppppppp rrrrrrrr xxxxxxxx *****
cc061 BBBBLLL jkkkkkkk ..... ssssssss j..... Pppppppp Pppppppp rrrrrrrr xxxxxxxx *****
cc071 AAAARRRR SSSS tttt MNNnaaaa xxxxxxxx DDDDDDDD DDDDDDDD JJJJJJJJ cccccccc
cc081 00000000 ..... EEEEEEEE ..... nnnnnnnh wwwwwwwv kkkkkkkk
cc091 ***** ***** ***** Pppppppp ..... zzzzzzzz mmmmmmmn UUUUUUUU
cc101 ..... ***** ***** nnnnnnnh ***** FFFFFFFF hhhhhhhh hhhhhhhh FFFFFFFF
cc111 ..... tttt FFFFFFFF GGGGcccc vvvvvvvv ..... llllllll ..... FFFFFFFF .....
cc121 kkkkkkkk ..... uooooUUU ssssssss ..... ssssssss bbbbbbbb ***** cccccccc
cci002 cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc
cci012 cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc
cci022 cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc cccccccc
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# **THE NEXT STAGE PROTOTYPE (NSP)**

# NSP Goals

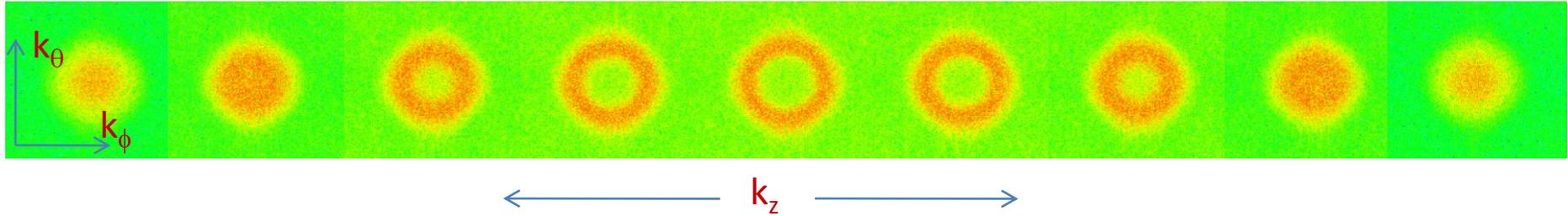
- Even with the FNAL computing cluster, testing future foreground algorithms will be time-consuming on the straw-man designs.
- An intermediate step would be to test the algorithms on a smaller scale prototype.
- This prototype should have these goals:
  - Demonstrate FFT beam forming
  - Understand calibration issues
  - Demonstrate foreground subtraction
  - Understand engineering costs
- A reasonable goal of the NSP would be to observe the first BAO peak

# BAO Signal First Peak from 400-1400MHz

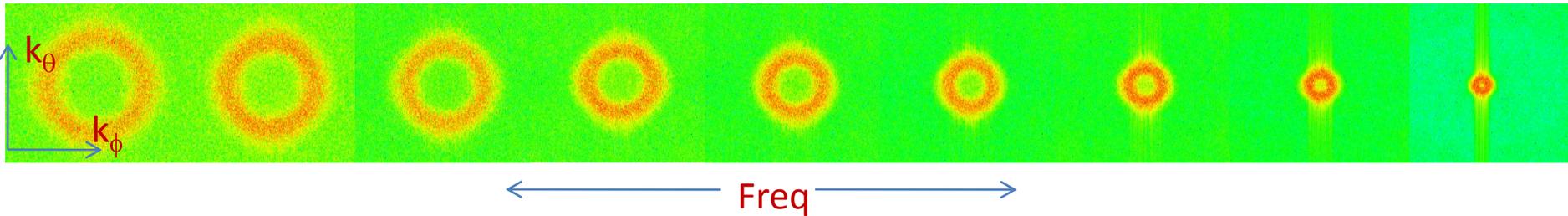


# BAO First Peak 3-D K space

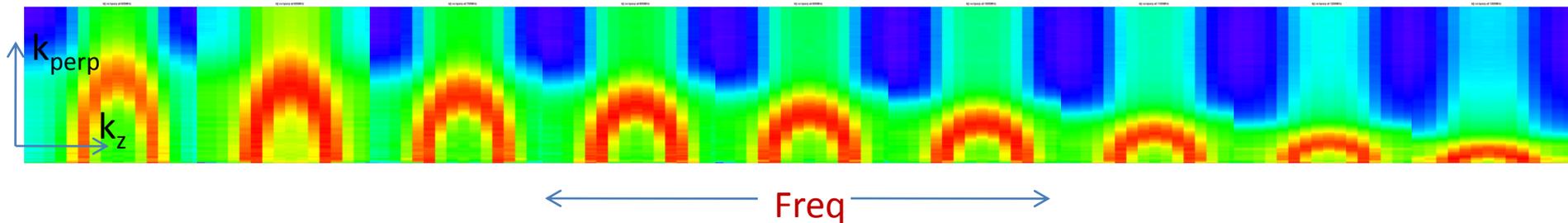
BAO First Peak in 3-D k-Space at 750 MHz – ResBw = 1/128 MHz



BAO First Peak from 500-1300MHz; Kperp at “ $k_{||} = 0$ ”; ResBW = 1/128 MHz



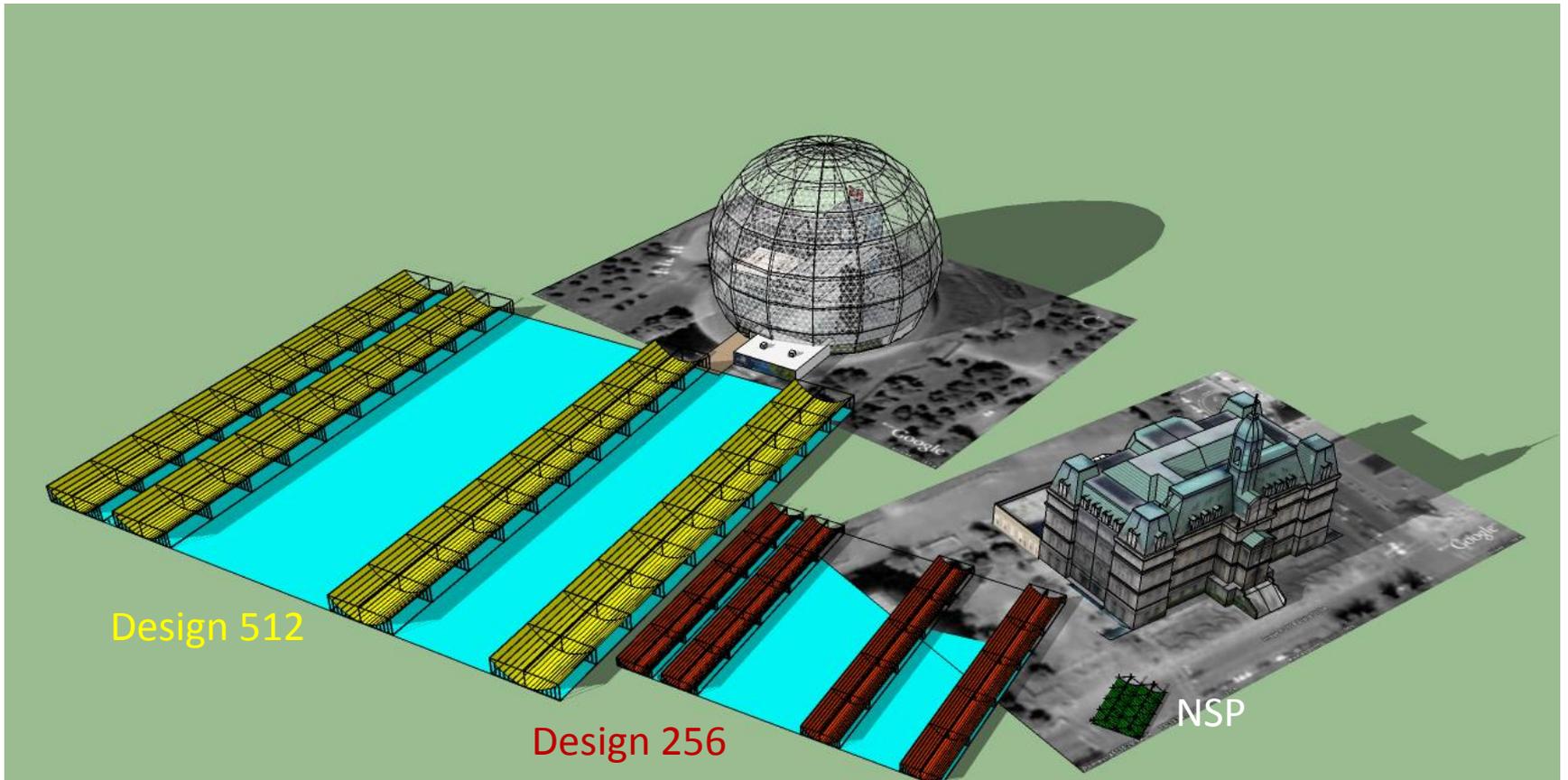
BAO First Peak from 500-1300MHz;  $k_{perp}$  vs “ $k_{||}$ ”; ResBW = 1/128 MHz



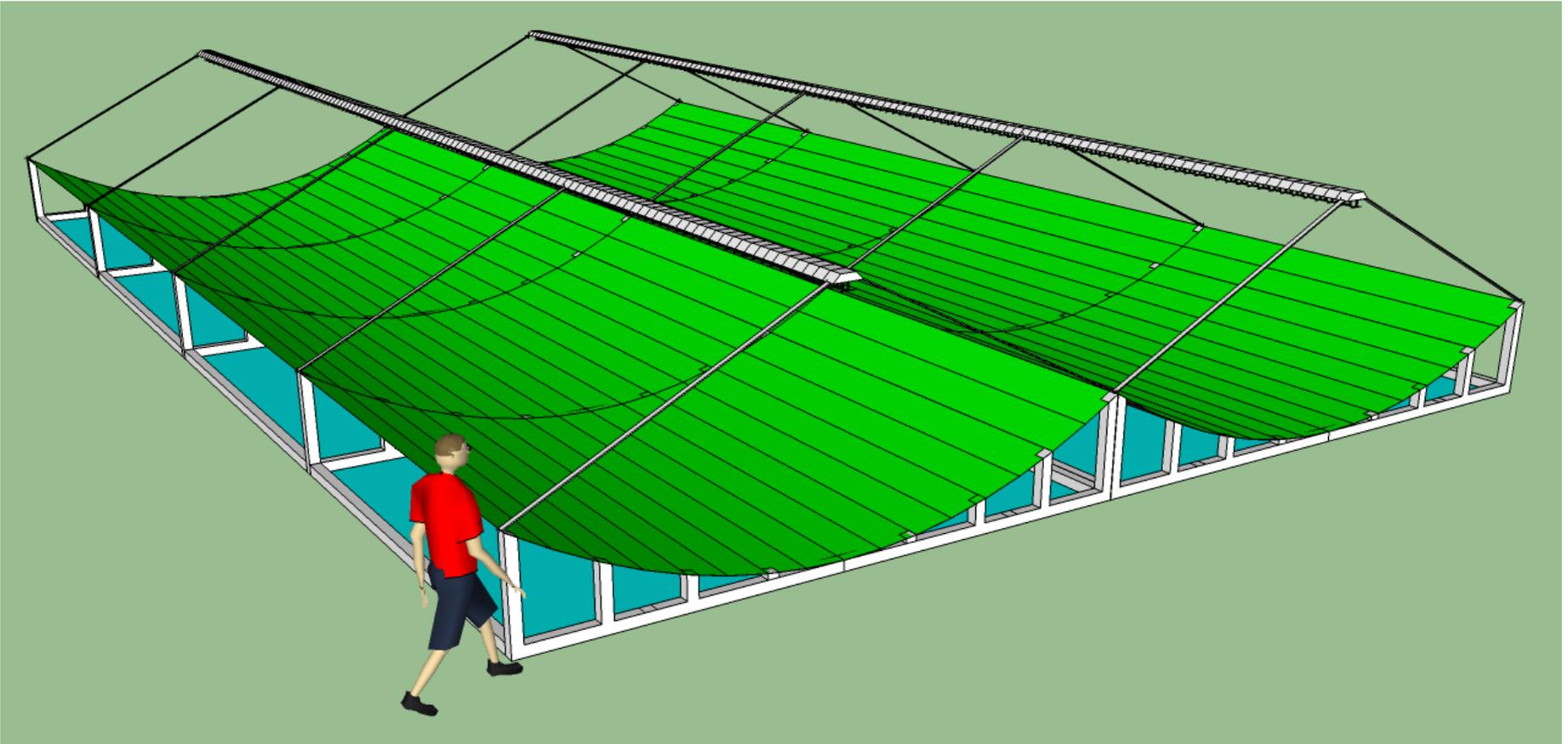
# NSP Parameters: A Proposal

- The NSP could have a center frequency of 1100 MHz
  - Will require less feeds per cylinder to resolve BAO than a lower frequency
  - might mitigate some problems with RFI
  - provide an overlap with optical observations of BAO.
- At this frequency
  - To resolve the crossover between the first and second BAO peak, the prototype will need to have a resolution of ~1.5 degrees.
    - 128 feeds per cylinder at a feed spacing of 0.135 meters (17 meters cylinder length)
    - The cylinder widths should be on the order of 5.0 meters
  - A bandwidth of 100 MHz with a resolution of 1 MHz is needed.

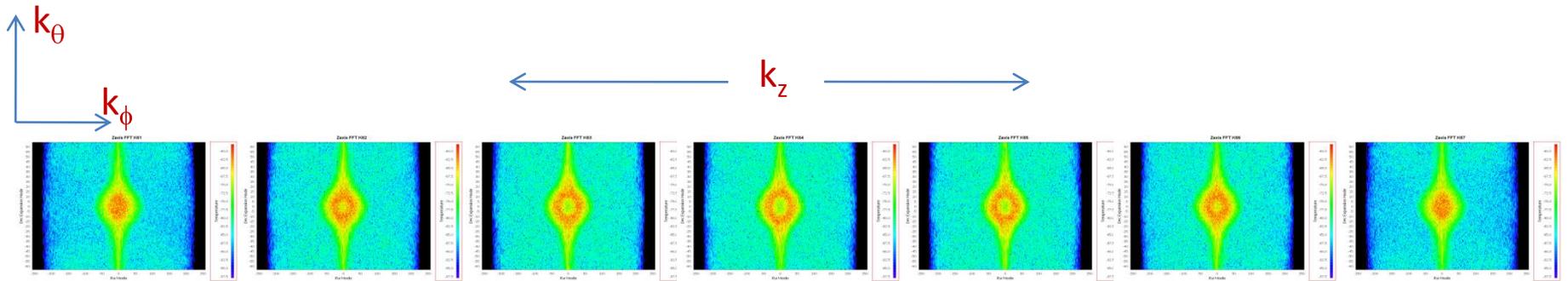
# NSP



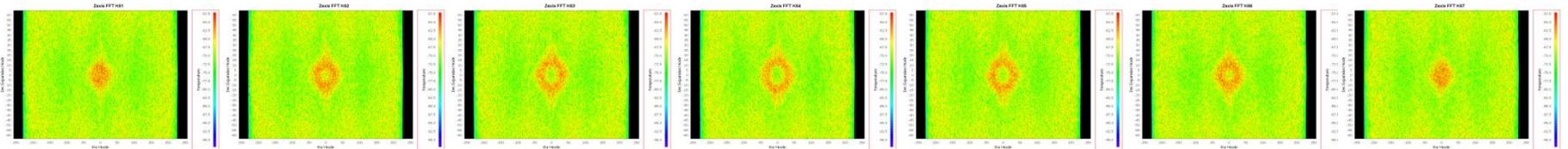
# NSP



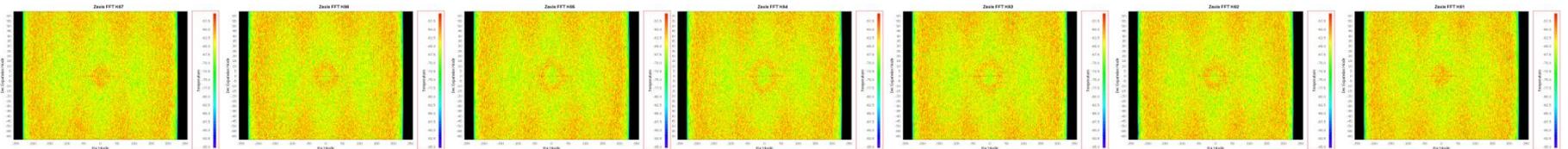
# Reconstructed BAO first Peak from NSP



$10^9$  Integration days

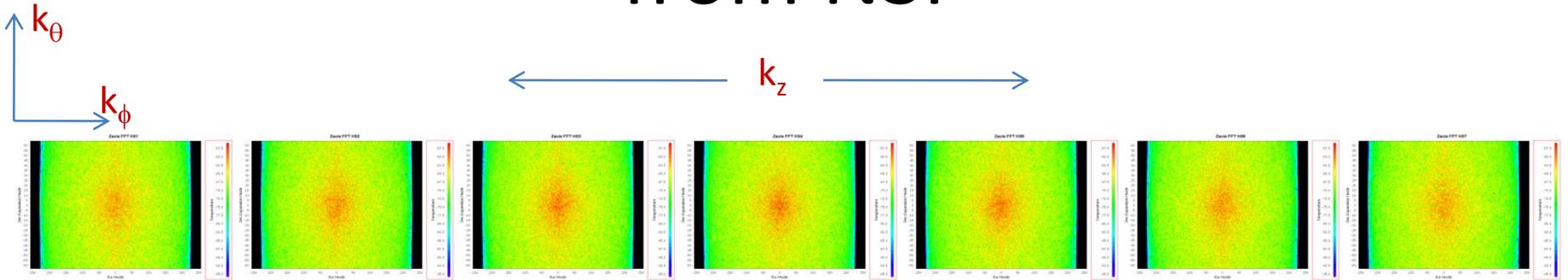


1000 Integration days

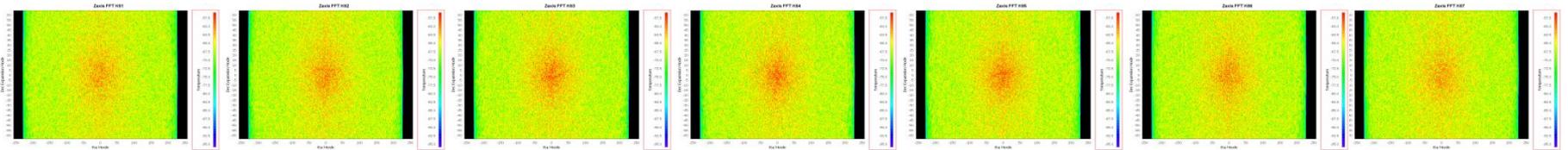


100 Integration days

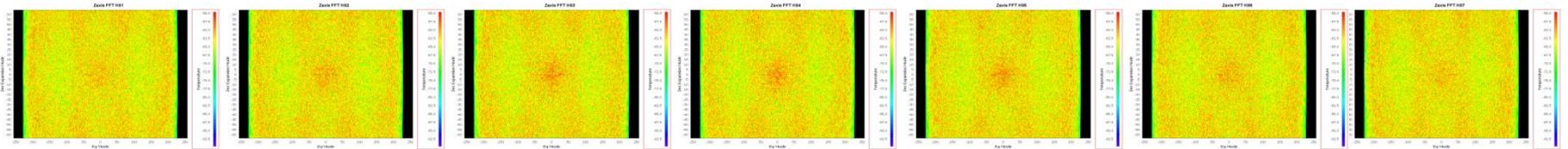
# Reconstructed Large Scale Structure from NSP



10<sup>9</sup> Integration days

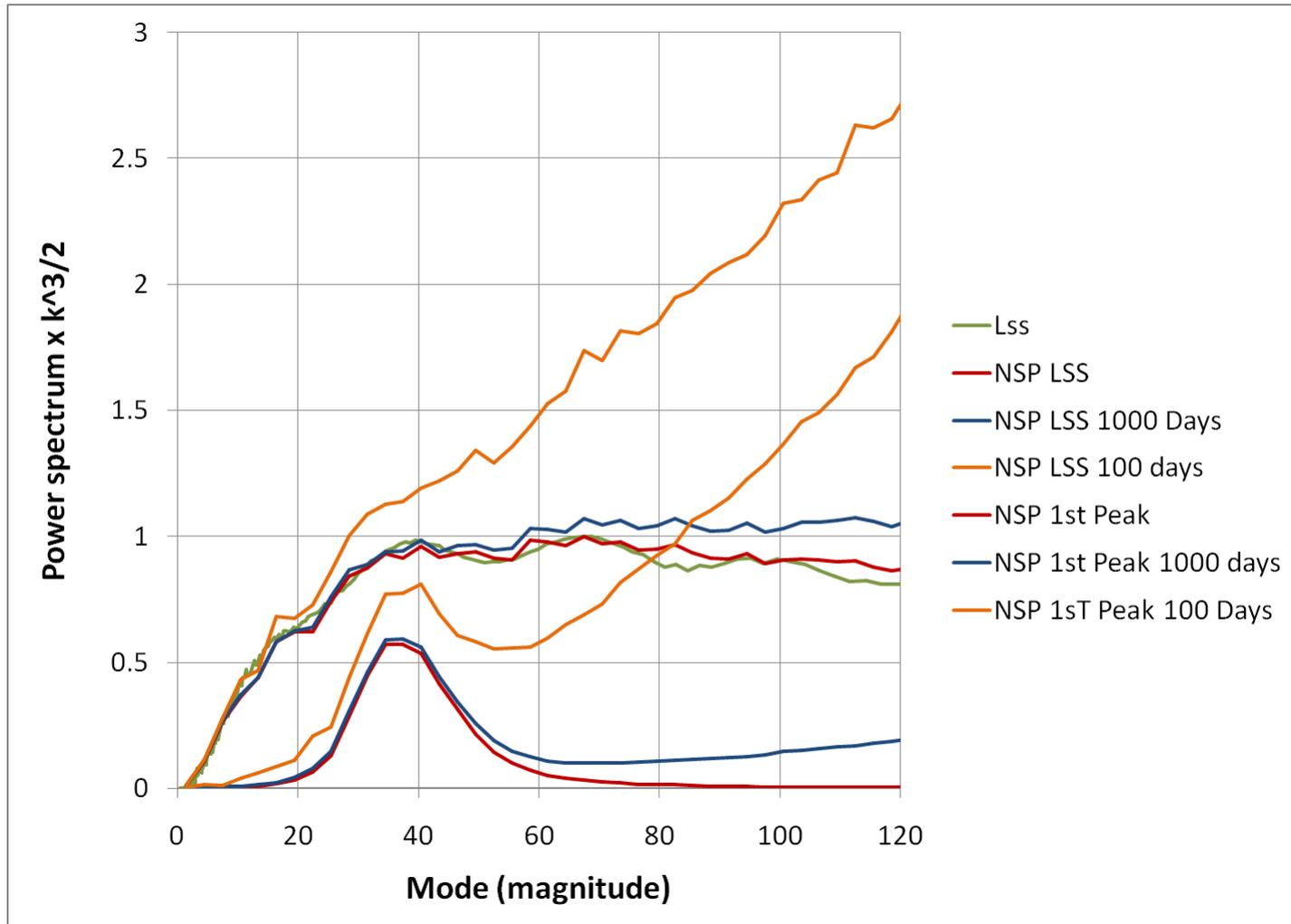


1000 Integration days



100 Integration days

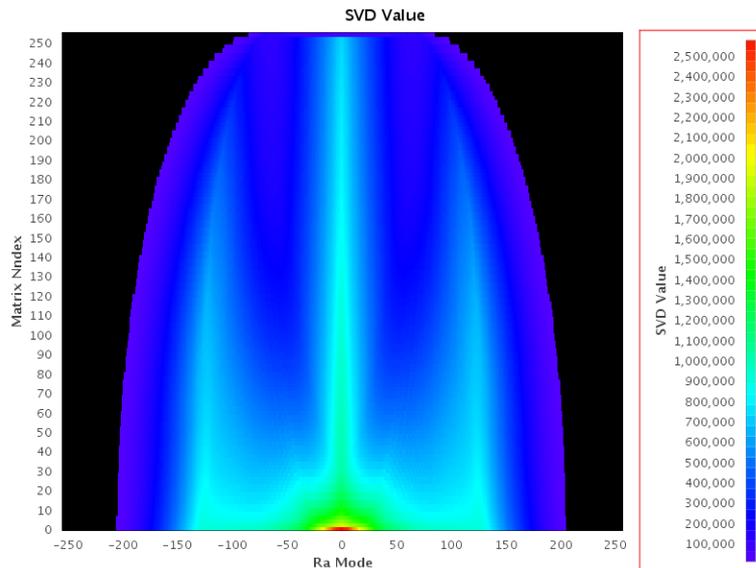
# NSP Noise Performance



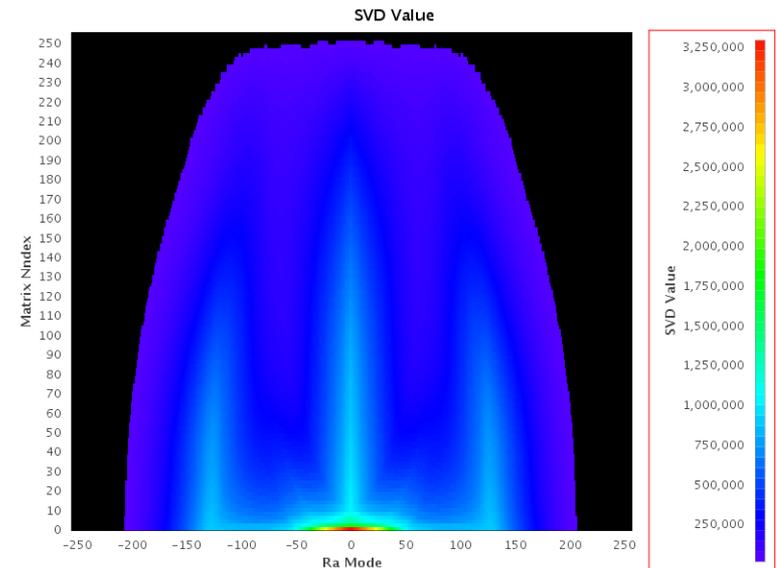
# Intermediate Step to Foreground Removal of a Reconstructed Sky

- I am currently having problems with how I handle the smoothness of the cuts on the singular value decomposition of the cylinder model matrix as a function of frequency.

$$\begin{aligned} & \left\{ \left\{ \left[ \hat{A}_m \right]^{(1)} \right\}^T \right\}^* [\tilde{V}_m]^{(1)} + \left\{ \left\{ \left[ \hat{A}_m \right]^{(2)} \right\}^T \right\}^* [\tilde{V}_m]^{(2)} \\ & = \left( \left\{ \left\{ \left[ \hat{A}_m \right]^{(1)} \right\}^T \right\}^* + \left\{ \left\{ \left[ \hat{A}_m \right]^{(2)} \right\}^T \right\}^* \right) [\tilde{T}_m] \end{aligned}$$



1100MHz



1113MHz

# Simple Demonstration of Foreground Removal of a Reconstructed Sky

- Separate issues of telescope modeling and sky reconstruction from foreground removal algorithms.
  - Assume that the issues with sky reconstruction algorithm will be solved.
- **Temporarily:** Use sky maps with frequency-smooth foregrounds overlaid on top of BAO signal to test foreground removal algorithms

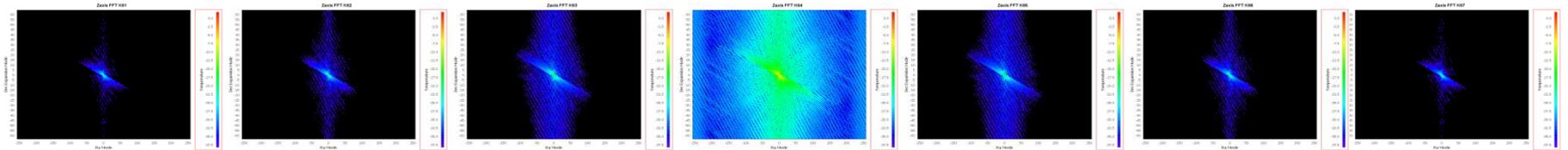
# Frequency-Smoothed Sky Algorithm in Reconstructed K-space

- Work in reconstructed sky transverse k-space
  - Addresses using up polynomial fitting “horsepower” on mode mixing
- Smooth in frequency by fitting an N order polynomial along frequency axis for each transverse k-space pixel
- Subtract frequency smoothed k-space from raw k-space
- Fourier transform along frequency axis
- Look the transverse k-space slices at high  $k_{||}$  mode number.

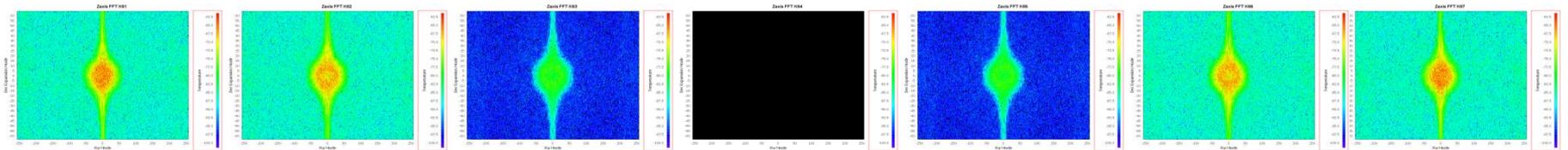
# BAO Signal+ Angelica Sky

$k_\theta$   
 $k_\phi$

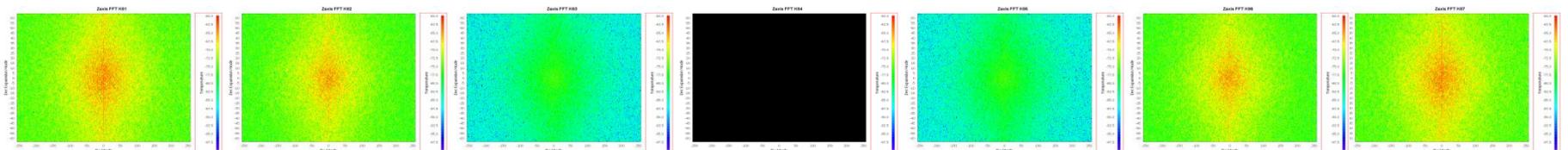
$k_z$



No filtering

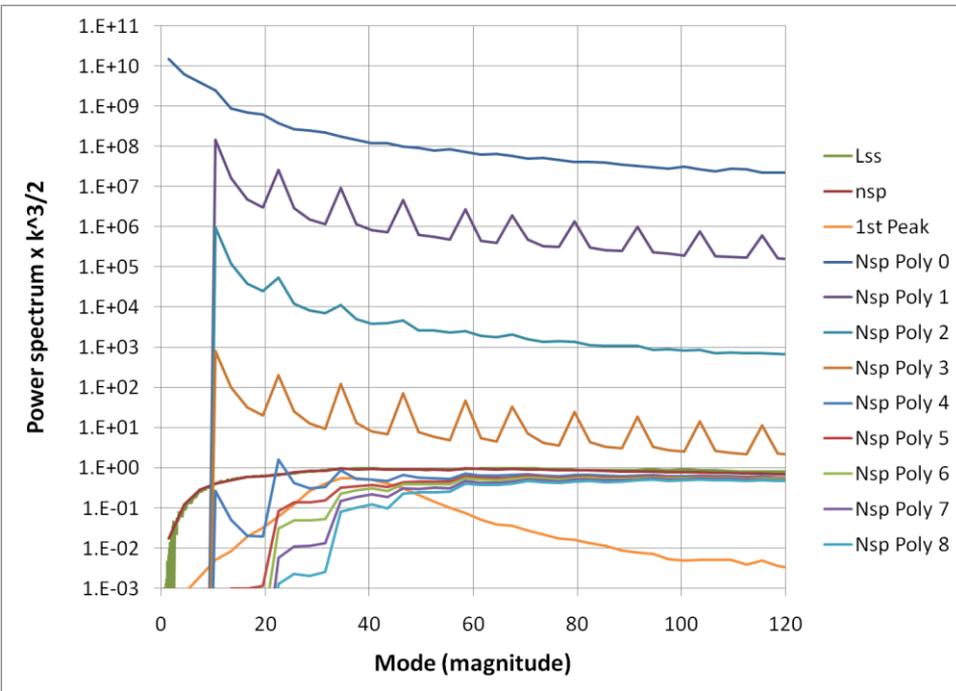


BAO first Peak + Angelica  
5<sup>th</sup> order Polynomial filter

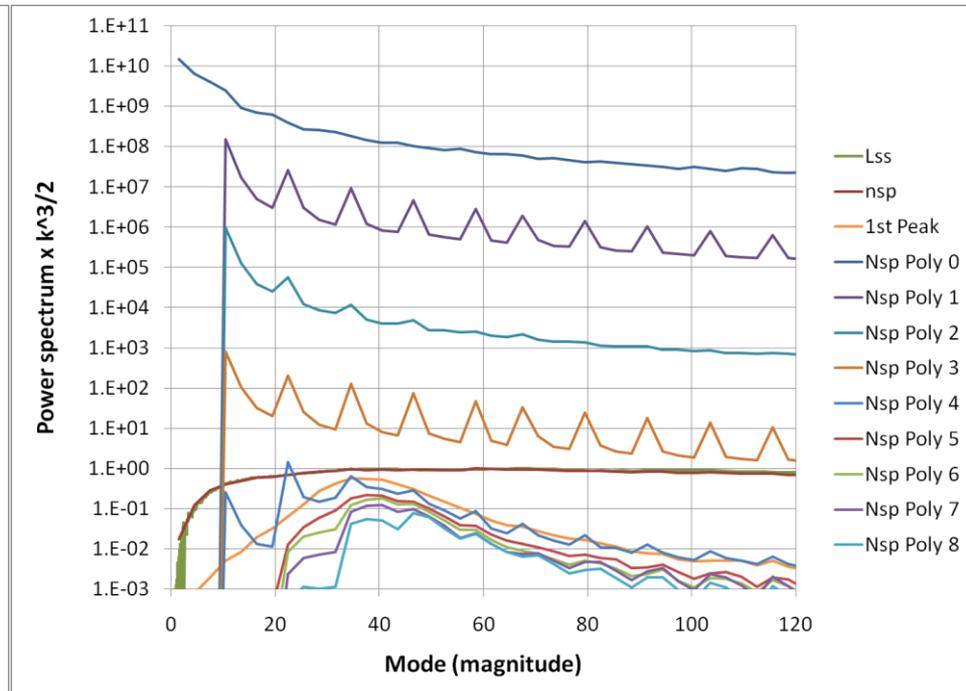


Large Scale Structure+ Angelica  
5<sup>th</sup> order Polynomial filter

# Polynomial Filter Effectiveness

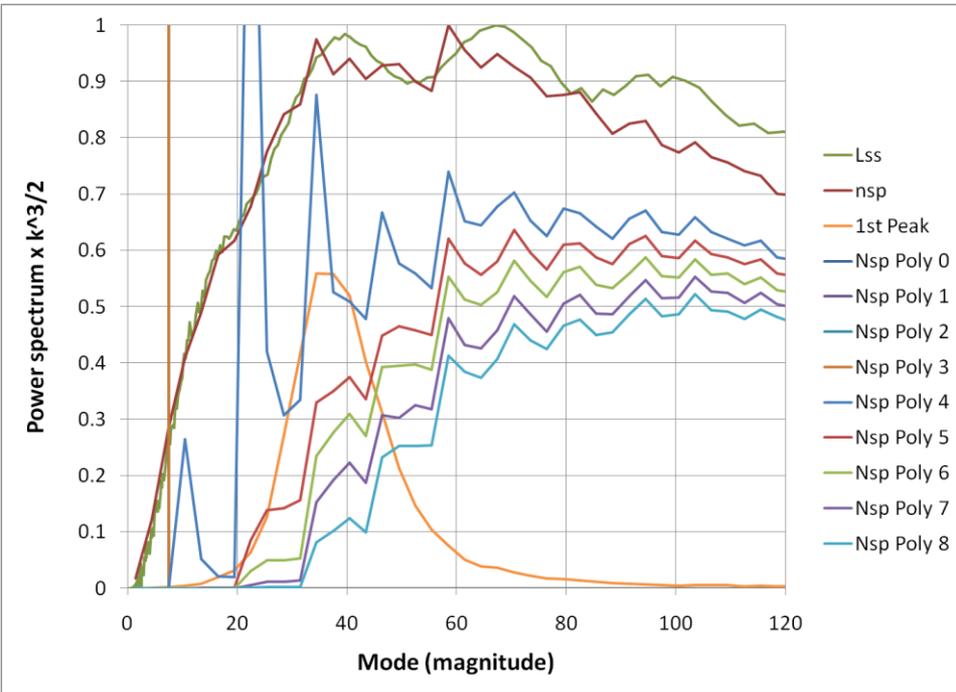


Large Scale Structure+ Angelica

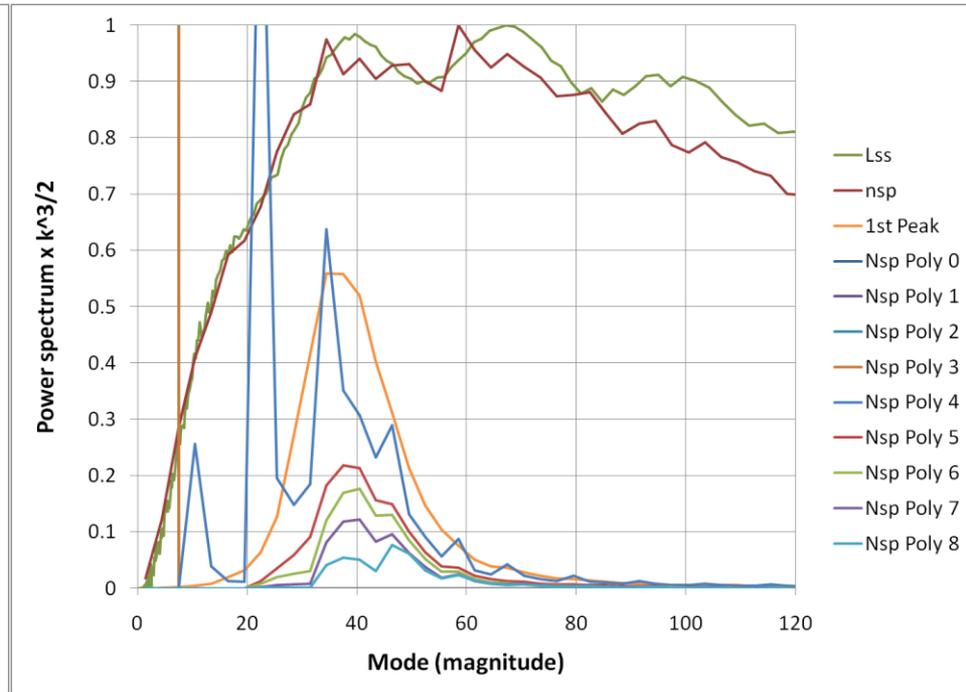


BAO first Peak + Angelica

# Polynomial Filter Effectiveness



Large Scale Structure+ Angelica



BAO first Peak + Angelica

# Summary

- We have developed a requirement methodology for evaluating Cylindrical Radio Telescope (CRT) Performance
- We presented a number of CRT designs that can achieve a Dark Energy Task Force Figure of merit of  $\sim 300$  for a cost of about 20M\$
- We have developed a end to end software simulation package that
  - Can mimic real world performance of the telescope
  - Explore the required accuracy of the telescope transfer function.
  - Determine the effects of sky “pathology”
  - Evaluate foreground removal algorithms